FITA SCIENTIFIC SEMINAR

ON THE INTUITIONISTIC FUZZY DATABASE: *Theories and Applications*

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Outline

- Introduction.
- Intuitionistic Fuzzy Set.
- Intuitionistic Fuzzy Relations.
- Intuitionistic Fuzzy Database.
- Conclusions.

1. Introduction

- Fuzzy set Theory introduced by L. A. Zadeh in 1965 [6], in which has important notion 'Fuzzy set and membership function':

  Let a set E be fixed. A Fuzzy Set (FS) A in E is an object having the form:

  $$A = \{<x, \mu_A(x)> \mid x \in E\}$$

  where $\mu_A : E \rightarrow [0, 1]$ *is called the membership function for the fuzzy set A.*

- A fuzzy database (FDB) based on similarity relation was introduced by Buckles and Petry is a generalization of the classical relational database [5].

1. Introduction *(cont.)*

- The Theory of *Intuitionistic Fuzzy Set* (IFS) introduced by Atanassov is a generalization of of the fuzzy set theory.
- IFS theory has been applied in different areas:
  - Logic programming [4]
  - Decision Making Problems [2]
  - Optimization Problem [1]
  - Medical Diagnosic [3], …
- In particular, IFS is used in the construction of Intuitionistic fuzzy databases, is based on Intuitionistic Fuzzy Relations (IFR).

2. Intuitionistic Fuzzy Set

- Definition 2.1
  - *Let a set E be fixed. An IFS A in E is an object having the form:*

$$A = \{ < x, \mu_A(x), \nu_A(x) > | x \in E \}$$

where $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ *define the degree membership and non-membership of the element x*$\in$E to the set A, and for every x $\in$ E then:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

  - *The amount* $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ *is called the hesitation part.*

- Example 2.1. Voting can be a good example of IFS. For each candidate x_i, voters may be divided into 3 group of those who: vote for, vote against and refusal of voting. Let E = $\{x_1, x_2, x_3\}$, then an IFS A in E may be:
  - A = $\{<x_1, 0.8, 0.1>, <x_2, 0.7, 0.2>, <x_3, 0.5, 0.4>\}$, or in other notation:
  - $A = \{\frac{(0.8,0.1)}{x_1}; \frac{(0.7,0.2)}{x_2}; \frac{(0.5,0.4)}{x_3}\}$

2. Intuitionistic Fuzzy Set *(cont.)*

- Definition 2.2
  - *If A and B are two IFS of the set E, then:*

$A \subset B$ iff $\forall x \in E$, $[\ \mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)\ ]$

$A = B$ iff $\forall\ x \in E$, $[\ \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)\ ]$

$\bar{A} = \{<x, \nu_A(x), \mu_A(x) > | x \in E\}$

$A \cap B = \{<x\ , \min(\mu_A(x)\ , \mu_B(x))\ , \max(\nu_A(x), \nu_B(x)) > | x \in E\}$

$A \cup B = \{<x\ , \max(\mu_A(x)\ , \mu_B(x))\ , \min(\nu_A(x), \nu_B(x)) > | x \in E\}$

  - *Obviously every fuzzy set A has the form:*

$$A = \{<x\ , \mu_A(x)\ , \mu_{A^c}(x) > | x \in E\}$$

- Example 2.2

Let $A = \{\frac{(0.8,0.1)}{x_1}; \frac{(0.7,0.2)}{x_2}; \frac{(0.5,0.4)}{x_3}\}$ *and* $B = \{\frac{(0.7,0.2)}{x_1}; \frac{(0.8,0.1)}{x_2}; \frac{(0.6,0.2)}{x_3}\}$ *then:*

- $A \cup B = \{\frac{(0.8,0.1)}{x_1}; \frac{(0.8,0.1)}{x_2}; \frac{(0.6,0.4)}{x_3}\}$, $A \cap B = \{\frac{(0.7,0.2)}{x_1}; \frac{(0.7,0.2)}{x_2}; \frac{(0.5,0.4)}{x_3}\}$
- $\bar{A} = \{\frac{(0.1,0.8)}{x_1}; \frac{(0.2,0.7)}{x_2}; \frac{(0.4,0.5)}{x_3}\}$.

3. Intuitionistic Fuzzy Relations

- Definition 3.1

Let X and Y be two sets. An Intuitionistic Fuzzy Relation (IFR) R from X to Y is an IFS on $X \times Y$, that characterized by membership function μ_R and non-membership function v_R.

An IFR R from X to Y denoted by R(X→Y) and defined by:

$$R = \{ <(x, y), \mu_R(x, y), \nu_R(x, y)> \mid x \in X, y \in Y \}$$

where: $\mu_R : X \times Y \to [0, 1]$ and $\nu_R : X \times Y \to [0, 1]$, *satisfy:* $\mu_R(x, y) + \nu_R(x, y) \le 1$

- Definition 3.2

Let Q(X→Y) and R(Y→Z) be two IFRs. The max-min-max composition $R \circ Q$ is the intuitionistic fuzzy relation from X to Z, defined by the membership function $\mu_{R \circ Q}$ and non-membership function $v_{R \circ Q}$:

$$\mu_{Q \circ R}(x, z) = \bigvee_y [\, \mu_Q(x, y) \wedge \mu_R(y, z) \,],\ \forall (x, z) \in X \times Z \text{ and } \forall y \in Y.$$

$$\nu_{Q \circ R}(x, z) = \bigwedge_y [\, \nu_Q(x, y) \vee \nu_R(y, z) \,],\ \forall (x, z) \in X \times Z \text{ and } \forall y \in Y.$$

3. Intuitionistic Fuzzy Relations *(cont.)*

- Definition 3.3
  - *An IFR $R(X \to X)$ (also called IFR R on X), is said to be:*

  (i) Reflexive: iff $\forall x \in X$, $\mu_R(x, x) = 1$,

  (ii) Symmetric: iff $\forall x_1, x_2 \in X$, $\mu_R(x_1, x_2) = \mu_R(x_2, x_1)$ and $\nu_R(x_1, x_2) = \nu_R(x_2, x_1)$.

  (iii) Transitive: iff $R^2 \subseteq R$, where $R^2 = R \circ R$

  - *An IFR R on X is called Intuitionistic Tolerance relation if R is reflexive and symmetric.*
  - *An IFR R on X is called Intuitionistic Similarity relation if R is reflexive and symmetric and transitive.*
- Definition 3.4

  Let A be an IFS of the set E. For $\alpha \in [0, 1]$, the α-cut of A is the scisp set A_α defined by:

$$A_\alpha = \{ x : x \in E, \text{ either } \mu_A(x) \geq \alpha \text{ or } \nu_A(x) \leq 1 - \alpha \}.$$

or:

$$A_\alpha = \{ x : x \in E, \nu_A(x) \leq 1 - \alpha \}.$$

(note that $\mu_A(x) \geq \alpha$ ensures $\nu_A(x) \leq 1 - \alpha$ but not conversely.)

3. Intuitionistic Fuzzy Relations *(cont.)*

- **Definition 3.5**

  Let T be an intuitionistic fuzzy tolerance relation on X, given $\alpha \in [0, 1]$, then:

  - *two elements $x, y \in X$ are α-similar iff $\nu_R(x, y) \leq 1-\alpha$, denoted by $xT_\alpha y$.*
  - *two elements $x, z \in X$ are α-tolerate iff either $xT_\alpha z$ or there exists a sequence $y_1, y_2, \ldots, y_r \in X$ such that $xT_\alpha y_1 T_\alpha y_2 \ldots y_r T_\alpha z$, denoted $x\, T_\alpha^+\, z$*

- **Lemma 3.1**

  - *If T be an intuitionistic fuzzy tolerance relation on X , then T_α^+ is an equivalence relation. For any $\alpha \in [0, 1]$, T_α^+ partitions X into equivalence classes.*
  - *If T be an intuitionistic fuzzy similarity relation on X , then T_α is an equivalence relation, for any $\alpha \in [0, 1]$.*

- **Lemma 3.2**

  If T be an intuitionistic fuzzy similarity relation on X , then for any $\alpha \in [0, 1]$, T_α and T_α^+ generate identical equivalence classes.

3. Intuitionistic Fuzzy Relations *(cont.)*

- Example 3.1
  - *Consider the Intuitionistic Fuzzy relation T on $X = \{x_1, x_2, x_3, x_4\}$, T =*

	x_1	x_2	x_3	x_4
x_1	(1, 0)	(.8, .1)	(.6, .2)	(.3, .4)
x_2	(.8, .1)	(1, 0)	(.4, .5)	(.5, .3)
x_3	(.6, .2)	(.4, .5)	(1, 0)	(.6, .3)
x_4	(.3, .4)	(.5, .3)	(.6, .3)	(1, 0)

  - *Easy to see that, T is an intuitionistic fuzzy tolerance relation.*
  - *It can be computed that:*

for α = 1.0, the partition of X determined by T_α^+ is: $\{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}\}$.

for α = 0.9, the partition of X determined by T_α^+ is: $\{\{x_1, x_2\}, \{x_3\}, \{x_4\}\}$.

for α = 0.8, the partition of X determined by T_α^+ is: $\{\{x_1, x_2, x_3\}, \{x_4\}\}$.

for α = 0.7, the partition of X determined by T_α^+ is: $\{\{x_1, x_2, x_3, x_4\}\}$.

4. Intuitionistic Fuzzy Database

- Definition 4.1
  - *An intuitionistic fuzzy database (IFDB) relation R is a subset of the cross product:*

$$2^{D_1} \times 2^{D_2} \times \cdots \times 2^{D_m},$$

  - *An intuitionistic fuzzy tuple of R has the form* $t_i = (d_{i1}, d_{i2}, \ldots, d_{im})$, *where* $d_{ij} \subseteq D_j$.
  - *For each domain* D_j, *given an IF tolerance relation* T_j, *defined by the membership function:*

$$\mu_{T_j} : D_j \times D_j \to [0, 1]$$

and non-membership function:

$$\nu_{T_j} : D_j \times D_j \to [0, 1]$$

4. Intuitionistic Fuzzy Database *(cont.)*

- Example 4.1
  - *Consider a relation scheme of VEHICLE (Type , Color , Price).*

- Domain(Type) = D_1 = {sportcar, sedan, bicycle, truck, motobike},
- Domain(Color)= D_2 = {Blue, Green, Red},
- Domain(Price)= D_3 = {Very Expensive, Expensive, Modest, Affordable, Cheap}

  - *Now consider the intuitionistic fuzzy tolerance relation T_2 on D_2.*
  - *We have IFDB on VEHICLE scheme with respect to intuitionistic fyzzy tolerance relation T_2, is given by:*

T_2	B	G	R
Blue (B)	(1, 0)	(0.7,0.2)	(0, 1)
Green (G)	(0.7, 0.2)	(1, 0)	(0, 1)
Red (R)	(0, 1)	(0, 1)	(1, 0)

Tuple #	Type	Color	Price
t_1	{sport car}	{Blue, Green}	{very expensive}
t_2	{sedan}	{Blue}	{modest}
t_3	{motorbike}	{Green}	{cheap}
t_4	{sedan}	{Red}	{afforable}
t_5	{sport car}	{Red}	{expensive}

4. Intuitionistic Fuzzy Database *(cont.)*

- Hypothetical case study for an application in the fight against crime
  - *Supose that one murder has taken place at an area in a deep, dark line.*
  - The police suspects that the murderer is also from the same area.
  - Listening to the eye-witness: murderer has *more or less full big hair coverage*, *more or less curly hair texture and he has moderately large build*.
  - Police refer to a data file of all the suspected criminals of the that area, the information table with attributes 'HAIR COVERAGE', HAIR TEXTURE' and 'BUILD' is named 'CRIMINAL DATA' and given as below:

* CRIMINAL DATA

NAME	HAIR COVERAGE	HAIR TEXTURE	BUILD
Arup	Full Small(FS)	Stc.	Large
Boby	Rec.	Wavy	Very Small(VS)
Chandra	Full Small(FS)	Straight(Str.)	Small(S)
Dutta	Bald	Curly	Average(A)
Esita	Bald	Wavy	Average(A)
Falguni	Full Big(FB)	Stc.	Very Large(VL)
Gautom	Full Small	Straight	Small(S)
Halder	Rec.	Curly	Average(A)

4. Intuitionistic Fuzzy Database *(cont.)*

Now, consider the intuitionistic fuzzy tolerance relations:

- Relation T_1 on the domain D_1 of attribute 'HAIR COVERAGE', where:

  D_1 = {FB, FS, Rec. , Bald}

- Relation T_2 on the domain D_2 of attribute 'HAIR TEXTURE', where:

  D_2= {Str., Stc., Wavy, Curly}

- Relation T_3 on the domain D_3 of attribute 'BUILD', where:

  D_3= {VL, L, A, S, VS}

	FB	FS	Rec.	Bald
FB	(1, 0)	(.8, .1)	(.4, .4)	(0, 1)
FS	(.8, .1)	(1, 0)	(.5, .4)	(0, .9)
Rec.	(.4, .4)	(.5, .4)	(1, 0)	(.4, .4)
Bald	(0, 1)	(0, .9)	(.4, .4)	(1, 0)

	Str.	Stc.	Wavy	Curly
Str.	(1, 0)	(.7, .3)	(.2, .7)	(.1, .7)
Stc.	(.7, .3)	(1, 0)	(.3, .4)	(.5, .2)
Wavy	(.2, .7)	(.3, .4)	(1, 0)	(.4, .4)
Curly	(.1, .7)	(.5, .2)	(.4, .4)	(1, 0)

	VL	L	A	S	VS
VL	(1, 0)	(.8, .2)	(.5, .4)	(.3, .6)	(0, 1)
L	(.8, .2)	(1, 0)	(.6, .4)	(.4, .5)	(0, .9)
A	(.5, .4)	(.6, .4)	(1, 0)	(.6, .3)	(.3, .6)
S	(.3, .6)	(.4, ..5)	(.6, .3)	(1, 0)	(.8, .2)
VS	(0, 1)	(0, .9)	(.3, .6)	(.8, .2)	(1, 0)

4. Intuitionistic Fuzzy Database *(cont.)*

- Now, the job is to find out a list of the Criminals who resemble with *more or less full big hair coverage*, *more or less curly hair texture* and *moderately large build*.
- The job can be done with a query on the intuitionistic fuzzy database . It can be translated into relational algebra in the following form:

  Select NAME, HAIR COVERAGE, HAIR TEXTURE, BUILD

  From CRIMINAL DATA

  With Level(HAIR COVERAGE)=0.8

  Level(HAIR TEXTURE) = 0.8

  Level (BUILD) = 0.7

  Where HAIR COVERAGE = 'Full Big'

  HAIR TEXTURE = 'Curly'

  BUILD = 'Large'

- ***Result:*** The above query gives rise to the following relation. And, further investigation now is to be done on this only, instead of dealing with a hugo list of criminal.

*LIKELY MURDERER

NAME	HAIR COVERAGE	HAIR TEXTURE	BUILD
{ Arup, Falguni }	{ Full Big, Full Small }	{ Curly, Stc. }	{Large, Very Large }

5. Conclusions

- Conclusions.
  - Presented an overview of the basic concepts of intuitionistic fuzzy set theory, intuitionistic fuzzy relations.
  - Introduced the concept of intuitionistic fuzzy databases and an application of this database
- Next Studies
  - Data Dependencies and Normalization of Intuitionistic Fuzzy Database.
  - The concepts of Picture Fuzzy Set and Picture Fuzzy Database.

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Your Questions ?

Thank you for
your attention !