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## NORMALIZATION IN A FUZZY RELATIONAL DATABASE MODEL

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### ABSTRACT

Fuzzy relational database models generalize the classical relational database model by allowing uncertain and imprecise information to be represented and manipulated. In this paper, we introduced fuzzy extensions of the normal forms for similarity based fuzzy relational database model. First of all we have designed an algorithm to find the fuzzy closure of attribute set which can be utilized to find fuzzy key. Next, with the concepts of  $\alpha$ -ffd, partial  $\alpha$ -ffd as in [1], we have defined different fuzzy normal forms in fuzzy relational database. We also include real life application to show how normalization based on  $\alpha$ -ffds of fuzzy relation is done.

### KEYWORDS

$\alpha$ -ffd , partial  $\alpha$ -ffd, fuzzy key, fuzzy closure of attribute set, fuzzy normal forms.

### 1. INTRODUCTION

It is well known that the classical relational data model introduced by Codd [2] in 1970 can handle only precise and exact data in an information source. However, information obtained from real life applications is more often imprecise and ambiguous. Fuzzy database models [3-24] based on the fuzzy set theory proposed by Zadeh [25] in 1965 have been extensively studied and cultivated in literature to deal with such uncertain and fuzzy information in relational databases.

One of the primary purposes of any databases is to decrease data redundancy and to provide data consistency. Data redundancies and insertion, deletion and update anomalies have also been of great concern in relational database. Like integrity constraints normalization process also plays an important role in designing a good relational database. Studies related to

normalization process minimizing redundancies and different anomalies for fuzzy relational model are accounted in [18-24].

In this chapter, our objective is to design a fuzzy relational database model by minimizing redundancies and also minimizing the insertion, deletion and update anomalies. For this first of all fuzzy closure of attribute set is presented to find fuzzy key. Next, we have introduced a number fuzzy normal forms such as fuzzy first (**F1NF**), fuzzy second (**F2NF**), fuzzy third (**F3NF**) and fuzzy BCNF (**FBCNF**) based on **ffds**. Finally, all these concepts are verified with an example.

The organization of the paper is as follows: The basics of fuzzy set theory have been reviewed in the following **section 2**. In **section 3**, we proposed an algorithm to find fuzzy closure of attribute set. **Section 4** discusses different fuzzy normalization techniques, namely **F1NF**, **F2NF**, **F3NF**, and **FBCNF** of a fuzzy relational database model by giving a real-life application. The final conclusions are reported in **section 5**.

## 2. BASIC DEFINITIONS

In this section, we first review some basic definitions from fuzzy set theory that will be useful throughout the paper and then define fuzzy functional dependency as introduced ( $\alpha$ -**ffd**) in [5] and revisit the basic propositions related to  $\alpha$ -**ffd**.

### 2.1 Basic Preliminaries on Fuzzy Set Theory

Fuzzy set theory, introduced by Zadeh [25] in 1965 has been widely used in the areas where we have to deal with imprecise or ambiguous data. Fuzzy set theory is a generalization of a crisp set theory. In this section we will explain some basic definitions on fuzzy set theory. Let  $U$  be a classical set of elements, called the universe of discourse. An element of  $U$  is denoted by  $u$ .

#### Definition 2.1.1

A fuzzy set  $F$  in a universe of discourse  $U$  is characterized by a membership function  $\mu_F : U \rightarrow [0,1]$  and  $F$  is defined as the set of ordered pairs  $F = \{(u, \mu_F(u)) : u \in U\}$ , where  $\mu_F(u)$  for each  $u \in U$  denotes the grade of membership of  $u$  in the fuzzy set  $F$ .

Note that a classical subset  $A$  of  $U$  can be viewed as a fuzzy subset with membership function  $\mu_A$  taken binary values, i.e.,

$$\begin{aligned} \mu_A &= 1 \text{ if } u \in A \\ &= 0 \text{ if } u \notin A \end{aligned}$$

The usual set theory operations such as union, intersection and complementation etc., have been extended to deal with fuzzy sets.

#### Definition 2.1.2

If  $A$  and  $B$  be two fuzzy sets of the universe of discourse  $U$ , then the fuzzy union of  $A$  and  $B$  is denoted by  $A \underset{\text{fuzzy}}{\cup} B$  and is defined as  $A \underset{\text{fuzzy}}{\cup} B = \{(x, \max\{\mu_A(x), \mu_B(x)\}) : x \in U\}$ .

**Definition 2.1.3**

If  $A$  and  $B$  be two fuzzy sets of the universe of discourse  $U$ , then the fuzzy intersection of  $A$  and  $B$  is denoted by  $A \underset{\text{fuzzy}}{\cap} B$  and is defined as  $A \underset{\text{fuzzy}}{\cap} B = \{ (x, \min \{ \mu_A(u), \mu_B(u) \}) : x \in U \}$

**Definition 2.1.4**

The complement of a fuzzy set  $F$  is denoted by  $F^c$  and is defined by the membership functions  $\mu_{F^c}(u)$  as follows:

$$\mu_{F^c}(u) = 1 - \mu_F(u) \forall u \in U .$$

**Definition 2.1.5**

A fuzzy set  $F$  is an **empty** fuzzy set, denoted by  $\phi$ , if and only if its membership function  $\mu_F(u) = 0$  for all  $u \in U$ .

**Definition 2.1.6**

A fuzzy set  $A$  is **contained** in another fuzzy set  $B$ , written as  $A \subseteq B$ , if and only if,  $\mu_A(u) \leq \mu_B(u) \forall u \in U$ .

**Definition 2.1.7**

Two fuzzy sets  $A$  and  $B$  are **equal**, written as  $A = B$ , iff,  $A \subseteq B$  and  $B \subseteq A$ , that is,  $\mu_A(u) = \mu_B(u) \forall u \in U$ .

**Definition 2.1.8**

Let  $U = U_1 \times U_2 \times \dots \times U_n$  be the Cartesian product of  $n$  universes, and  $F_i, i = 1, 2, \dots, n$  be fuzzy sets in their corresponding universe of discourses  $U_i, i = 1, 2, \dots, n$  respectively. Also, let  $u_i \in U_i, i = 1, 2, \dots, n$ . Then the Cartesian product  $F = F_1 \times F_2 \times \dots \times F_n$  is defined to be a fuzzy set of  $U = U_1 \times U_2 \times \dots \times U_n$  with the membership function defined as follows:

$$\mu_F(u_1, u_2, \dots, u_n) = \min(\mu_{F_1}(u_1), \mu_{F_2}(u_2), \dots, \mu_{F_n}(u_n)) .$$

**Definition 2.1.9**

Mathematically an  $n$ -ary fuzzy relation  $r$  is a fuzzy subset of the Cartesian product of  $n$  universes. Thus for given  $n$  universes  $U_1, U_2, \dots, U_n$ , a fuzzy relation  $R$  is a fuzzy subset of  $U_1 \times U_2 \times \dots \times U_n$  and is characterized by the  $n$ -variate membership function  $\mu_R : U_1 \times U_2 \times \dots \times U_n \rightarrow [0, 1]$ .

## 2.2 Fuzzy Functional Dependency:

Next, to introduce the new notion of **ffd** as defined in [5], we give the following definitions and terminologies.

Let  $X$  be a universal set and  $\mathfrak{R}$  be a fuzzy tolerance relation on  $X$ . Consider a choice parameter  $\alpha \in [0,1]$  to be predefined by the database designer.

**Definition 2.2.1:**  $(\alpha)_{\mathfrak{R}}$ -nearer or  $\alpha$ -nearer elements.

Two elements  $x_1, x_2 \in X$  are said to be  $(\alpha)_{\mathfrak{R}}$ -nearer or  $\alpha$ -nearer if  $\mu_{\mathfrak{R}}(x_1, x_2) \geq \alpha$ . We denote this by the notation  $x_1 N_{(\alpha)_{\mathfrak{R}}} x_2$ .

**Definition 2.2.2:**  $(\alpha)_{\mathfrak{R}}$ -equality or  $\alpha$ -equality elements.

$x_1, x_2 \in X$  are said to be  $(\alpha)_{\mathfrak{R}}$ -equal or  $\alpha$ -equal if

- 1) either  $x_1 N_{(\alpha)_{\mathfrak{R}}} x_2$  or
- 2)  $\exists y_1, y_2, y_3, \dots, y_{r-1}, y_r \in X$  such that  
 $\{x_1 N_{(\alpha)_{\mathfrak{R}}} y_1, y_1 N_{(\alpha)_{\mathfrak{R}}} y_2, y_2 N_{(\alpha)_{\mathfrak{R}}} y_3, \dots, y_{r-1} N_{(\alpha)_{\mathfrak{R}}} y_r\}$  and  $y_r N_{(\alpha)_{\mathfrak{R}}} x_2$ .

This equality is denoted by the notation  $x_1 E_{(\alpha)_{\mathfrak{R}}} x_2$ .

**Definition 2.2.3:**  $\delta_{(\alpha)_{\mathfrak{R}}}$  relation on  $X$ .

The crisp relation  $\delta_{(\alpha)_{\mathfrak{R}}}$  on  $X$  is defined as: For  $x_1, x_2 \in X$ ,  $x_1 \delta_{(\alpha)_{\mathfrak{R}}} x_2$  if  $x_1 E_{(\alpha)_{\mathfrak{R}}} x_2$ .

Now, consider a relation  $r(R)$  of a relation schema  $R(A_1, A_2, \dots, A_n)$ . Let us assume a fuzzy tolerance relation  $\mathfrak{R}_i$  on the domain  $dom(A_i) \forall i=1, 2, \dots, n$ . Let  $\mathfrak{R}$  denote the set  $\{\mathfrak{R}_1, \mathfrak{R}_2, \dots, \mathfrak{R}_n\}$  of fuzzy tolerance relations. Let  $X = \{X_1, X_2, \dots, X_k\} \subseteq R$ . We now define  $(\alpha)_{\mathfrak{R}}$ -equality of two tuples  $t_1[X]$  and  $t_2[X]$  in a relational database design.

**Definition 2.2.4:**  $(\alpha)_{\mathfrak{R}}$ -equality of  $t_1[X]$  and  $t_2[X]$ .

Two tuples  $t_1[X]$  and  $t_2[X]$  are said to be  $(\alpha)_{\mathfrak{R}}$ -equal if  $t_1[X_i] E_{(\alpha)_{\mathfrak{R}_i}} t_2[X_i], \forall i=1, 2, \dots, k$ .

The equality notation is denoted as  $t_1[X] \mathcal{E}_{(\alpha)_{\mathfrak{R}}} t_2[X]$  or simply by  $t_1[X] \mathcal{E}_{\alpha} t_2[X]$ .

**Definition 2.2.5:** Fuzzy Functional Dependency (**ffd**).

Let  $X, Y \subset R = \{A_1, A_2, \dots, A_n\}$ . Choose a parameter  $\alpha \in [0,1]$  and propose a fuzzy tolerance relation  $\mathfrak{R}$ . A fuzzy functional dependency (**ffd**), denoted by  $X \xrightarrow{(\alpha)_{\mathfrak{R}}} Y$  or simply by  $X \xrightarrow{\alpha} Y$ , is said to exist, if whenever  $t_1[X] \mathcal{E}_{\alpha} t_2[X]$ , it is also the case that  $t_1[Y] \mathcal{E}_{\alpha} t_2[Y]$ .

This **ffd** can be read as “ $X$  fuzzy functionally determines  $Y$  at  $\alpha$ -level of choice” or “ $Y$  fuzzy functionally depends on  $X$  at  $\alpha$ -level of choice” and is called an  $\alpha$ -**ffd**. Clearly, by definition of  $\alpha$ -**ffd**, it follows that for any subset  $X$  of  $R$  and for any  $\alpha \in [0,1]$ ,  $X \xrightarrow{\alpha} Y$ .

Also we have the following straight forward propositions:

**Proposition 2.2.1:** The relation  $\delta_{(\alpha)_\mathfrak{R}}$  defined on  $X$  is an **equivalence relation** (See ref. [1])

**Proposition 2.2.2:** For any tuple  $t$  and for any  $\alpha \in [0,1]$ ,  $t[X] \varepsilon_\alpha t[X]$ .

**Proposition 2.2.3:**  $t_1[X] \varepsilon_{\alpha_1} t_2[X] \Rightarrow t_2[X] \varepsilon_{\alpha_1} t_1[X]$ .

**Proposition 2.2.4:** If  $0 \leq \alpha_2 \leq \alpha_1 \leq 1$ , then  $t_1[X] \varepsilon_{\alpha_1} t_2[X] \Rightarrow t_1[X] \varepsilon_{\alpha_2} t_2[X]$ .

**Proposition 2.2.5:** If  $0 \leq \alpha_2 \leq \alpha_1 \leq 1$ , then  $X \xrightarrow{\alpha_1} Y \Rightarrow X \xrightarrow{\alpha_2} Y$ .

**Proposition 2.2.6:** The **ffd**  $X \xrightarrow{1} Y \Rightarrow$  classical fd  $X \rightarrow Y$ .

**Proposition 2.2.7:** The **ffd**  $X \xrightarrow{0} Y \Rightarrow$  “ $X$  does not functionally determine  $Y$ ”

### 2.3 Inference rules for $\alpha$ -ffd

Consider a relational schema  $R(A_1, A_2, \dots, A_n)$  and  $X, Y, Z, W \subseteq R$ , then we have the following set of inference rules similar to classical functional dependency defined for fuzzy functional dependency of a fuzzy relational database. The rules (R2.3.1) to (R2.3.6) are called fuzzy **Armstrong’s axioms**.

(R2.3.1)  $\alpha$ -ffd reflexive rule: If  $Y \subset X$ , then  $X \xrightarrow{\alpha} Y$ .

(R2.3.2)  $\alpha$ -ffd augmentation rule: If  $X \xrightarrow{\alpha} Y$ , then  $XZ \xrightarrow{\alpha} YZ$ .

(R2.3.3)  $\alpha$ -ffd transitive rule: If  $X \xrightarrow{\alpha_1} Y$  and  $Y \xrightarrow{\alpha_2} Z$ , then  $X \xrightarrow{\min(\alpha_1, \alpha_2)} Z$ .

(R2.3.4)  $\alpha$ -ffd decomposition rule: If  $X \xrightarrow{\alpha} YZ$ , then  $X \xrightarrow{\alpha} Y$  and  $X \xrightarrow{\alpha} Z$ .

(R2.3.5)  $\alpha$ -ffd union rule: If  $X \xrightarrow{\alpha_1} Y$  and  $X \xrightarrow{\alpha_2} Z$ , then  $X \xrightarrow{\min(\alpha_1, \alpha_2)} YZ$ .

(R2.3.6)  $\alpha$ -ffd pseudo transitive rule: If  $X \xrightarrow{\alpha_1} Y$  and  $WY \xrightarrow{\alpha_2} Z$ , then  $WX \xrightarrow{\min(\alpha_1, \alpha_2)} Z$ .

### 2.4 Partial fuzzy functional dependency (partial $\alpha$ -ffd)

Partial fuzzy functional dependency (**partial  $\alpha$ -ffd**) for fuzzy relational database is defined below:

#### Definition 2.4.1

$Y$  is called partially fuzzy functionally dependent on  $X$  at  $\alpha$ -level of choice, ie,  $X \xrightarrow{\alpha} Y$  partially, iff  $X \xrightarrow{\alpha} Y$  and there exists a non empty set  $X' \subset X$ , such that,  $X' \xrightarrow{\alpha} Y$ . The concept of partial **ffd** then expresses the fact that after removal of an attribute  $A$  from  $X$ , the dependency still holds i.e., for an attribute  $A \in X$ ,  $X - \{A\}$  still fuzzy functionally determines  $Y$  at  $\alpha$ -level of choice.

### 2.5 fuzzy Key

Extending the idea of classical key in the fuzzy environment we have defined fuzzy key in [1] as follows:

#### Definition 2.5.1

Let  $K \subseteq R$  and  $F$  be a set of **ffds** for  $R$ . Then,  $K$  is called a **fuzzy key** of  $R$  at  $\alpha$ -level of choice where  $\alpha \in [0,1]$  iff  $K \xrightarrow{\alpha} R \in F$  and  $K \xrightarrow{\alpha} R$  is not a partial **ffd**.

### 3. FUZZY CLOSURE OF ATTRIBUTE SET

To find the fuzzy key of a relation scheme  $R$ , we should find the fuzzy closure of an attribute or set of attributes. Fuzzy closure of attribute set  $X$  denoted by  $X^+$  is the set of attributes which are fuzzy functionally determined by the attributes  $X$ . If the closure set  $X^+$  is the minimal set which contains all the attributes of the relation scheme  $R$  then  $X$  is called fuzzy key of the relation  $R$ . Below we explain an algorithm to find the fuzzy closure of an attribute or a set of attributes.

**Algorithm 3.1:** Computation of fuzzy closure of an attribute or a set of attributes.

**Input:** A fuzzy relational scheme  $R$ , a set of **ffds**  $F$  on  $R$  and an attribute or a set of attributes  $X$ .

**Output:** A set of attributes  $X^+$ , the fuzzy closure set of  $X$ .

**Method:**

Let  $X^+ = X$ .

i.e.,  $X^+ = (X, \alpha_1)$  where  $\alpha_1 = 1$  [ $\because X \xrightarrow{1} X$ ]

**repeat**

$X^* = X^+$ .

**for each ffd**  $Y \xrightarrow{\alpha_2} Z$  in  $F$

**do**

if  $Y \subseteq X^+$  then  $X^+ = (X^+ \cup Z, \alpha_3)$  where  $\alpha_3 = \min(\alpha_1, \alpha_2)$ .

$\alpha_1 = \alpha_3$ .

**end for**

**until**  $(X^* = X^+)$ .

**Example 3.1:** Let  $R(A, B, C, D)$  be a relational scheme and a set of **fds** and **ffds**  $F$  on  $R$  is given as  $F = \{A \xrightarrow{0.8} C, B \xrightarrow{0.75} D, C \xrightarrow{0.9} B\}$ . Find closure of  $A$ .

**Solution:**

Let  $X^+ = A$  i.e.,  $X^+ = (A, 1)$

**Repeat Case-I**

$X^* = (A, 1)$

for  $A \xrightarrow{0.8} C$

$X^+ = (AC, .8)$  [ $\because A \subseteq X^+$ ]

for  $B \xrightarrow{0.75} D$

$X^+ = (AC, .8)$  [ $\because B \not\subseteq X^+$ ]

for  $C \xrightarrow{0.9} B$

$X^+ = (ABC, .8)$  [ $\because C \subseteq X^+$ ]

Since  $X^* \neq X^+$  follow the repeat case again

**Repeat Case-II**

$$X^* = (ABC, .8)$$

$$\text{for } A \xrightarrow{0.8} C$$

$$X^+ = (ABC, .8) [\because A \subseteq X^+]$$

$$\text{for } B \xrightarrow{0.75} D$$

$$X^+ = (ABCD, 0.75) [\because B \subseteq X^+]$$

$$\text{for } C \xrightarrow{0.9} B$$

$$X^+ = (ABCD, 0.75) [\because C \subseteq X^+]$$

Since  $X^* \neq X^+$  again perform the repeat case

**Repeat Case-II**

$$X^* = (ABCD, 0.75)$$

$$\text{for } A \xrightarrow{0.8} C$$

$$X^+ = (ABCD, 0.75) [\because A \subseteq X^+]$$

$$\text{for } B \xrightarrow{0.75} D$$

$$X^+ = (ABCD, 0.75) [\because B \subseteq X^+]$$

$$\text{for } C \xrightarrow{0.9} B$$

$$X^+ = (ABCD, 0.75) [\because C \subseteq X^+]$$

Repeat case will stop since  $X^* = X^+$ .

Hence the fuzzy closure of  $A$  i.e.,  $X^+ = (ABCD, 0.75)$  which means  $A \xrightarrow{0.75} ABCD$ .

**4. FUZZY NORMALIZATION**

An important problem of the relational database design is how to obtain relation schemes in which the so called storage anomalies [26-28] are avoided as much as possible. Storage anomalies occur during updating operations and cause the inconsistency of data. In order to avoid storage anomalies as much as possible, Codd [26] introduced a series of normal forms, such as first (1NF), second (2NF), third (3NF) and BCNF normal forms.

For good database design any relation must follow at least 3NF normal form but BCNF, a stricter form of 3NF is most desirable normal form from the point of view of redundancy, so in fuzzy relational database, we also discuss the normalization techniques for fuzzy relation schemes called fuzzy normalization up to fuzzy BCNF normal forms. Following we will discuss step by step different normalization forms of fuzzy relational database say, fuzzy first (F1NF), fuzzy second (F2NF), fuzzy third (F3NF) and fuzzy BCNF (FBCNF) normal forms. To discuss the above theory we consider the fuzzy relation scheme  $EMP(Name, City, City\_Status, Experience, Salary)$  given below in Table-I with a set **fds** and **ffds**  $F$  as follows:

$$F = \{City \xrightarrow{0.99} City\_Status, Experience \xrightarrow{0.9} Salary, NameCity \rightarrow Experience\}$$

Name	City	City_Status	Experience	Salary
Smith	Delhi	30	13	more or less 50000
David	Kolkata	20	13.6	50000
John	Delhi	around 30	23	95000
Smith	Mumbai	10	23	95050

**Table-I:** A fuzzy relational instance  $r$  of  $EMP$  relation

The **fd**  $NameCity \rightarrow Experience$  can be express in **ffd** as  $NameCity \xrightarrow{1} Experience$ . So the above  $F$  set can be rewritten as follows:

$F = \{City \xrightarrow{0.99} City\_Status, Experience \xrightarrow{0.9} Salary, NameCity \xrightarrow{1} Experience\}$ . Next we have to find the fuzzy key of the relation  $EMP$ . Using **Algorithm 3.1** the fuzzy closure of  $NameCity$  is obtained as

$(NameCity)^+ = (Name \ City \ City\_Status \ Experience \ Salary, .9)$  which means  $NameCity \xrightarrow{0.9} Name \ City \ City\_Status \ Experience \ Salary$ . So, we can say,  $NameCity$  is the fuzzy key of  $EMP$  relation at **0.9**-level of choice.

#### 4.1 Fuzzy Prime and Non-prime Attributes

To be able to state the condition for fuzzy **2NF**, it is also necessary to define fuzzy prime and non-prime attributes for a relation.

##### Definition 4.1.1

Let  $A \in R$  and  $K$  be a fuzzy key set for  $R$ .  $A$  is called fuzzy prime attributes if and only if  $A \in K$ . Those attributes which are not fuzzy prime are called fuzzy non-prime.

For an attribute to be fuzzy prime attribute, it should be a part of at least one of the fuzzy candidate keys of the relation. Similarly, for an attribute to be a fuzzy non-prime attribute, it should not appear in any of the fuzzy candidate keys of the relations.

##### Example 4.1.1

Let us consider the above  $EMP(Name, City, City\_Status, Experience, Salary)$  relation and **ffd** set  $F = \{City \xrightarrow{0.99} City\_Status, Experience \xrightarrow{0.9} Salary, NameCity \rightarrow Experience\}$  of **Table-I**.

Here  $NameCity$  is a fuzzy key at **0.9**-level of choice. Therefore, we conclude that the attributes  $Name$  and  $City$  are fuzzy prime attributes at **0.9**-level of choice and attributes  $City\_Status$ ,  $Experience$  and  $Salary$  are fuzzy non-prime attributes.

#### 4.2 Fuzzy First Normal Form

The first one of the classical normal forms that is extended and generalized within the framework of  $\alpha$ -equality based fuzzy relational model is the **1NF**.



**Definition 4.2.1**

Let  $D_k$  be the domain of attributes  $A_k$ , a relation schema  $R$  is called to be in first fuzzy normal form i.e., **F1NF** if and only if for any relation  $r$  in  $R$ , none of the attributes contained multi-valued.

**Example 4.2.1**

**Table-I** defined above is in **F1NF**.

**4.3 Fuzzy Second Normal Form**

The fuzzy second normal form, **F2NF**, is based on the concept of the full **ffd**. By using the concepts of fuzzy key and partial fuzzy functional dependence, we can define the **F2NF**.

**Definition 4.3.1**

Let  $F$  be the set of **ffds** for relation schema  $R$  and  $K$  be a fuzzy key of  $R$  at  $\alpha$ -level of choice.  $R$  is called to be in fuzzy second normal form i.e., **F2NF**, if and only if for none of the nonprime attributes is partially fuzzy functionally dependent on the fuzzy key.

**Example 4.3.1**

Let us consider the *EMP* relation and **ffd** set  $F$  of **Table-I**. i.e.,  $EMP(Name, City, City\_Status, Experience, Salary)$  and

$$F = \{City \xrightarrow{0.99} City\_Status, Experience \xrightarrow{0.9} Salary, NameCity \rightarrow Experience\}.$$

**Fuzzy key:** *NameCity* at **0.9**-level of choice.

Here we have the nonprime attributes *City\\_Status* which is fuzzy partially dependent on fuzzy key *NameCity* at **0.9**-level of choice. So, **Table-I** is not in **F2NF**.

To satisfy **F2NF** we have to decomposed the above **Table-I**. Since **ffd**  $City \xrightarrow{0.99} City\_Status$  is the only **ffd** violating the **F2NF** condition, so the two **F2NF** decomposed relations (using the decomposition concept for classical relational database) are

*EMPCityStatus(City, City\\_Status)* with **ffds**

$$F1 = \{City \xrightarrow{0.99} City\_Status\}$$

**Fuzzy Key:** *City* at **0.99**-level of choice.

and

*EMP\\_INFO(Name, City, Experience, Salary)* with **ffds**

$$F2 = \{Experience \xrightarrow{0.9} Salary, NameCity \rightarrow Experience\}$$

**Fuzzy Key:** *NameCity* at **0.9**-level of choice.

#### 4.4 Fuzzy Third Normal Form (F3NF)

The normalization process takes a relation schema through a series of tests to certify whether it satisfies a certain normal form. The process proceeds in a top-down fashion. In a database design satisfying the fuzzy third normal form i.e., **F3NF**, insertion, deletion and update anomalies will be minimum.

##### Definition 4.4.1

Let  $F$  be the set of **ffds** for relation schema  $R$  and  $K$  be a fuzzy key of  $R$  at  $\alpha$ -level of choice.  $R$  is called to be in fuzzy third normal form i.e., **F3NF**, if and only if  $R$  is in **F2NF** and  $R$  should not contain any **ffd** among fuzzy nonprime attributes i.e., for any non-trivial **ffd**  $X \xrightarrow{\alpha} A$  in  $F$  either  $X$  contains the fuzzy key or  $A$  is fuzzy-prime.

##### Example 4.4.1

From this definition of **F3NF** we can say that the above relation *EMPCityStatus* is in **F3NF**, but relation *EMP\_INFO* is not in **F3NF**. In relation *EMP\_INFO* the **ffd**  $Experience \xrightarrow{0.9} Salary$  is violating the **F3NF** condition. So, we decompose the relation *EMP\_INFO* into **F3NF** relations as follows:

*EMPExpSal(Experience, Salary)* with **ffds**

$F3 = \{ Experience \xrightarrow{0.9} Salary \}$

**Fuzzy Key:** *Experience* at **0.9**-level of choice.

*EMP\_MAIN\_INFO(Name, City, Experience)* with **ffds**

$F4 = \{ NameCity \rightarrow Experience \}$

**Fuzzy Key:** *NameCity* at **1**-level of choice. Here fuzzy key is also the classical key.

#### 4.5 Fuzzy Boyce Codd Normal Form (FBCNF)

Like its classical counterpart, fuzzy boyce codd normal form i.e., **FBCNF**, is a stricter form of fuzzy **3NF**. **FBCNF** ensures that there is no redundancy that can be detected using **ffd** information alone. It is more desirable normal form from the point of view of redundancy. The formal definition of the **FBCNF** can be given as follows.

##### Definition 4.5.1

Let  $F$  be the set of **ffds** for relation schema  $R$  and  $K$  be a fuzzy key of  $R$  at  $\alpha$ -level of choice.  $R$  is called to be in fuzzy BCNF normal form i.e., **FBCNF**, if and only if  $R$  is in **F3NF** and for any non-trivial **ffd**  $X \xrightarrow{\alpha} A$  in  $F$ ,  $X$  must be a fuzzy key or super fuzzy key of  $R$  i.e.,  $K \subseteq X$ .

##### Example 4.5.1

The following decomposed relations satisfying **F3NF** are also in **FBCNF**.

$EMPCityStatus(City, City\_Status), F1 = \{City \xrightarrow{0.99} City\_Status\}$

$EMPExpSal(Experience, Salary), F3 = \{Experience \xrightarrow{0.9} Salary\}$

$EMP\_MAIN\_INFO(Name, City, Experience), F4 = \{NameCity \rightarrow Experience\}$

## 5. CONCLUSION

Like the classical relational databases, fuzzy relational database is also being suffered from data redundancy and different anomalies if it is not designed properly. Fuzzy normalization based on  $\alpha$ -**ffd** plays an important role in order to design a good fuzzy relational database.

In this paper, firstly we have design an algorithm to find fuzzy closure of attribute set which helps in finding fuzzy key and then normalization process of fuzzy relation has been discussed by defining different fuzzy normal forms i.e., **F1NF**, **F2NF**, **F3NF**, and **FBCNF**. We have also introduced the definition of fuzzy prime and fuzzy nonprime attributes in order to state the condition for fuzzy normal forms. We illustrate how these fuzzy normal forms can be used to decompose an un-normalized fuzzy relation into a set of normalized relations with an example.

As further study involving the dependency preservation and lossless join property of fuzzy relational database, fuzzy join dependency, fuzzy inclusion dependency and related normal forms has been ongoing.

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