Chapter 3 : Limits and Continuity

Phan Quang Sang

Vietnam National University of Agriculture

Hanoi, 29 août 2016
http ://www.vnua.edu.vn/khoa/fita/pqsang/

Sommaire

Informal definition of Limits

The limit of $f(x)$, as x approaches c, is equal to L means that $f(x)$ can be made arbitrarily close to L whenever x is sufficiently close to c. We denote this by

$$\lim_{x \to c} f(x) = L.$$

If L is a finite number, we say that the limit exists and that $f(x)$ converges to L as x tends to c.
Some Examples :

$$\lim_{x \to 1} (x^2 + 1), \quad \lim_{x \to 2} \frac{x^2 - 4}{x - 2}.$$

Note that we choose x is very close to x but not equal to c.
We do not simply plug c into $f(x)$.

One sided limits

$\lim_{x\to c^+} f(x) = L$ when x approaches c from the right.
$\lim_{x\to c^-} f(x) = L$ when x approaches c from the left.

Examples : Find $\lim_{x\to 0^+} \frac{|x|}{x}$, $\lim_{x\to 0^-} \frac{|x|}{x}$

Example : consider $\lim_{x \to 0} \frac{1}{x^2}$.

Example : consider $\lim_{x \to 0} \frac{1}{x^2}$.

Remark

$\lim_{x \to c} f(x) = +\infty$ if $f(x)$ increases without bound as $x \to c$.
$\lim_{x \to c} f(x) = -\infty$ if $f(x)$ decreases without bound as $x \to c$.

Examples :

$$\lim_{x \to 2^+} \frac{1}{x-2} = +\infty,$$

$$\lim_{x \to 2^-} \frac{1}{x-2} = -\infty.$$

Limit at infinity

Some Examples :

$$\lim_{x\to\infty} \frac{x}{x+1},$$

$$\lim_{x\to\infty} \frac{2x^2-x+1}{x^2-x+1},$$

$$\lim_{x\to+\infty} \sqrt{x^2+3x-1}-\sqrt{x^2+1}.$$

Limit Laws

See the page 124

3.1.3. Problems page 127

The sandwich theorem

Theorem

If $f(x) \leq g(x) \leq h(x)$ for x close to c and if

$$\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L,$$

then

$$\lim_{x \to c} g(x) = L.$$

Example : show that

$$\lim_{x \to 0} x \sin \frac{1}{x} = 0.$$

A fundamental trigonometric limit

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

Example : show that

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 1.$$

Problems

Problems

3.1.3 Problems page 127

3.3.1 Problems page 142

3.4.1 Problems page 148

Home work
Exs 9, 18, 26, 31, 34, 48 page 127

Exs 5, 11, 15,27 page 142

Exs 4,13 page 148

Definition

A function f is said to be continuous at c if

$$\lim_{x \to c} f(x) = f(c).$$

Definition

A function f is said to be continuous at c if

$$\lim_{x \to c} f(x) = f(c).$$

That means,

- f is well defined at c ;
- $\lim_{x \to c} f(x)$ exists ;
- $\lim_{x \to c} f(x) = f(c)$.

If not, the function f is discontinuous at c.

Some examples

Definition

A function f is said to be continuous at c if

$$\lim_{x \to c} f(x) = f(c).$$

That means,

- f is well defined at c ;
- $\lim_{x \to c} f(x)$ exists ;
- $\lim_{x \to c} f(x) = f(c)$.

If not, the function f is discontinuous at c.

Some examples

Read yourself Def. page 131.

Properties of continuous functions

The Intermediate Value Theorem (I.V.T.)

Let f be a continuous functions on the closed interval $[a, b]$. For L is any real number between $f(a)$ and $f(b)$, then there exists at least one number $c \in (a, b)$ such that $f(c) = L$.

Example : let

$$f(x) = 3 + sinx \text{ for } 0 \leq x \leq \frac{3\pi}{2}.$$

Show that there exists at least one point c in $(0, \frac{3\pi}{2})$ such that $f(x) = \frac{5}{2}$.

Self study

1. 3.5.2. A final Remark on continuous function ;
2. The Bisection method : example 2 page 150.

Problems

Examples 2, 3, 4,5 page 130-132 : remove discontinuities (discontinuity removed ?)

3.2.3 Problems page 137

3.5.3 Problems page 152

Problems

Examples 2, 3, 4,5 page 130-132 : remove discontinuities (discontinuity removed ?)

3.2.3 Problems page 137

3.5.3 Problems page 152

Home work
Exs 8, 11, 16, 22, 27, 31, 37 page 137

Exs 3,4 page 152

THANK YOU FOR YOUR ATTENTION !