Chapter 7 (Math17B) Integration techniques and Computational methods

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Hanoi, 27 mars 2015

Contents

Example : differentiating the function $f(x) = sin(3x^2 + 1)$ by using the chain rule and after we reverse the procedure interm of indefinite integrals $\int cos(3x^2 + 1).\ 6xdx$.
Then we generalize the integral of the example to integrals of the form

$$I = \int f(g(x))g'(x)dx.$$

If F is an antiderivative of $f(x)$, then by using the chain rule we can easily check that

$$\frac{d}{dx}F(g(x)) = F'(g(x))g'(x).$$

We can therefore have

$$I = F(g(x)) + C,\ \ C \in \mathbb{R}.$$

Theorem

Let $u = g(x)$, *then*

$$I = \int f(g(x))g'(x)dx = \int f(u)du = F(u) + C = F(g(x)) + C. \tag{1.1}$$

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Some examples (on blackboard)

(1) $\int (2x+1)e^{x^2+x+1}dx$.

(2) $\int \frac{1}{x.\ln x}dx$.

(3) $\int (x+1)\sqrt{x-1}dx$.

Due to FTC II and by applying the substitution rule we get

$$\int_a^b f(g(x))g'(x)dx = F(g(x))\big|_a^b = \int_{g(a)}^{g(b)} f(u)\, du. \qquad (1.2)$$

Recall that we need to check whether the integrant is continuous over the integration interval.
There are two ways to calculate the integral : in terms of variable x, or u. But don't forget to change the limits of integration in the second way.

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(1) $\int_0^1 (2x+1)e^{x^2+x+1}dx$. (let $u = x^2 + x + 1$)

(2) $\int_1^2 \frac{3x^2+1}{x^3+x}dx$. (let $u = x^3 + x$, result $\ln 5$)

(3) $\int_{1/2}^1 \frac{1}{x^2}e^{\frac{1}{x}}dx$. (let $u = \frac{1}{x}$, result $e^2 - e$)

Let $u = u(x)$, $v = v(x)$ be differentiable functions, and then differentiating with respect to x we get

$$(uv)' = u'v + uv',$$

and so

$$uv' = (uv)' - u'v.$$

Integrating both sides of the above equation with respect to x, we have

$$\int uv'dx = \int (uv)'dx - \int u'vdx = uv - \int u'vdx.$$

Summarizing the last we get the 'Integration by parts rule' (IBP) :

Theorem

If $u = u(x)$, $v = v(x)$ are differentiable functions, then

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx, \qquad (2.3)$$

or in the short form

$$\int udv = uv - \int vdu.$$

Some examples (on blackboard).

Example

(1) $\int xsinx \ dx$.

(2) $\int xlnx \ dx$. (result $\frac{1}{2}x^2lnx + C$)

(3) Compute the Definite integral $\int_0^1 xe^{-x}dx$. (set $u = x$, $v' = e^{-x}$; result $1 - 2e^{-1}$)

(4) $\int lnx \ dx$ (result $xlnx - x + C$)

(5) $\int_0^1 x^2e^x \ dx$ (using IBP repeatedly, result $e - 2$)

(6) $\int e^x \cos x \ dx$ (using IBP repeatedly, result $\frac{1}{2}e^x(\cos x + \sin x + C)$)

In Problems 1-30, evaluate the integrals using integration by parts.

THANK YOU VERY MUCH !