Vietnam National University of Agriculture
Faculty of Information Technology
Department of Mathematics

The school year 2014-2015

Final exam, Subject No. 01
Course Syllabus math 17B- NH05005
Duration 90 minutes

Unauthorized materials

Date June , 2015

Exercise 1 (3 points): Use any method to integrate each of the following integrals

a) (1 p.) $\int_0^{\left(\frac{\pi}{2}\right)^2} sin\sqrt{x}\, dx$; b) (2 p.) $\int \frac{x^4+3x^2-3x+2}{x^3-x^2} dx.$

Exercise 2 (3 points): Considering the definite integral $\int_0^2 (x^2 - x)dx.$

a) (0,5 p.) Give a partition of the integration interval $[0,2]$ into n equal subintervals.
b) (1.5 p.) Find the Riemann sum for the integrand on $[0,2]$, associated to the above partition and right-hand endpoints.
c) (0,5 p.) Evaluate the limit of the Riemann sum as n goes to infinite to give the value of the integral.
d) (0.5 p.) Draw the graph of the function $f(x) = x^2 - x$ on $[0,2]$ and give a geometric meaning of the integral.

Exercise 3 (2 points): Show that the following improper integral is convergent $\int_1^{+\infty} \frac{1}{\sqrt[3]{x^4+1}} dx.$

Exercise 4 (2 points): Solve the differential equation $(x^2 + 1)y' = xy.$

Vietnam National University of Agriculture
Faculty of Information Technology
Department of Mathematics

The school year 2014-2015

Final Exam, Subject No. 02
Course Syllabus Math 17B- NH05005
Duration 90 minutes

Unauthorized materials

Date June , 2015

Exercise 1 (3 points): Use any method to integrate each of the following itegrals

a) (1 p.) $\int_0^{\left(\frac{\pi}{2}\right)^2} cos\sqrt{x}\,dx$; b) (2 p.) $\int \frac{x^4+4x^2+2x-1}{x^3+x^2}dx$

Exercise 2 (3 points): Considering the definite integral $\int_0^1 (x^2+2x)dx$.

a) (0,5 p.) Give a partition of the integration interval $[0,1]$ into n equal subintervals.
b) (1 p.) Find the Riemann sum for the integrand on $[0,1]$, associated to the above partition and right-hand endpoints.
c) (1 p.) Evaluate the limit of the Riemann sum as n goes to infinite to give the value of the definite integral.
d) (0,5 p.) Draw the graph of the function $f(x) = x^2 + 2x$ on $[0,1]$ and give a geometric meaning of the definite integral.

Exercise 3 (2 points): Show that the following improper integral is divergent $\int_1^{+\infty} \frac{1}{\sqrt[3]{x^2+1}}dx$.

Exercise 4 (2 points): Solve the differential equation $y' = y^2 sin\,x$.

Exercise		Brief solution of the Final Exam, Subject No. 01 Course Syllabus Math 17B- NH05005 Duration 90 minutes- Date June , 2015	Point
1 **(3 p.)**	a)	Use the substitution $t = \sqrt{x}$, hence $dx = 2tdt$	0.25
		Change the limits of integration $x = 0$, so $u = 0$; $x = \left(\frac{\pi}{2}\right)^2$, so $u = \frac{\pi}{2}$;	0.25
		The initial integral becomes $2\int_0^{\frac{\pi}{2}} t\ sint\ dt$	0.25
		By IBP we get $$2\int_0^{\frac{\pi}{2}} t\ sint\ dt = -2\int_0^{\frac{\pi}{2}} t\ d(cos\ t)\ dt$$	0.25
		$$= -2\left[[t\ cos\ t]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} cos\ t\ dt\right]$$	0.25
		. $$= -2\left[[t\ cos\ t]_0^{\frac{\pi}{2}} - [sin\ t]_0^{\frac{\pi}{2}}\right] = 2.$$	0.25
	b)	Making the long division $\frac{x^4+3x^2-3x+2}{x^3-x^2} = x + 1 + \frac{4x^2-3x+2}{x^3-x^2}$,	0.25
		and partial fraction: $\frac{x^4+3x^2-3x+2}{x^3-x^2} = x + 1 + \frac{1}{x}$	0.25
		$$-\frac{2}{x^2}$$	0.25
		$$+\frac{3}{x-1}$$	0.25
		Hence the integral is equal to $\frac{x^2}{2} + x + ln\|x\| + \frac{2}{x} + 3ln\|x-1\| + C$, for $C \in \mathbb{R}$.	0.25 0.25
2 **(3 p.)**	a)	The width of a subinterval is $\Delta_k = \frac{2-0}{n} = \frac{2}{n}$,	0.25
		hence the partition is given by the subintevals $[x_k, x_{k+1}]$, where $x_0 = 0, x_1 = 1.\frac{2}{n}, x_2 = 2.\frac{2}{n}, \dots, x_k = k.\frac{2}{n}, \dots, x_n = n.\frac{2}{n} = 2.$	0.25
	b)	Choosing $c_k = x_k = k.\frac{2}{n}, k = 1, \dots, n,$	0.25
		then the Riemann sum associated is $$\sigma_n = \frac{2}{n}\sum_{k=1}^{n}(x_k^2 - x_k) = \frac{2}{n}\left[\sum_{k=1}^{n}\left(k.\frac{2}{n}\right)^2 - \sum_{k=1}^{n} k.\frac{2}{n}\right]$$	0.25
		$$= \frac{2}{n}\left[\frac{4}{n^2}\sum_{k=1}^{n} k^2 - \frac{2}{n}\sum_{k=1}^{n} k\right]$$	0.25
		$$= \frac{2}{n}\left[\frac{4}{n^2}\frac{n(n+1)(2n+1)}{6} - \frac{2}{n}\frac{n(n+1)}{2}\right]$$	0.25 0.25
		$$= \frac{2(n^2+3n+2)}{3n^3}.$$	0.25

	c)	$\lim_{n\to\infty} \sigma_n = \frac{2}{3}$,	0.25
		hence the integral is $\frac{2}{3}$.	0.25
	d)	Draw the graph of the function $f(x) = x^2 - x$ on $[0,2]$.	0.25
		A geometric meaning of the definite integral: its value, $\frac{2}{3}$ is the area of the region enclosed by the graph of f, the x-axis , and the vertical line $x = 2$.	0.25
3 **(2 p.)**		For every $x \geq 1$ we have $x^4 + 1 > x^4 > 0$, and so $$0 < \frac{1}{\sqrt[3]{x^4+1}} \leq \frac{1}{\sqrt[3]{x^4}} = x^{-4/3}.$$	0.25 0.25
		On the other hand the impoper integral $\int_1^{+\infty} x^{-4/3} dx = \left[\frac{x^{-1/3}}{-1/3}\right]_1^{\infty} = 3$ is convergent,	1
		therefore the given integral is also convergent by comparison.	0.5
4 **(2 p.)**		It is clear that $y \equiv 0$ is a particular solution of the eqution. (1)	0.25
		If $y \neq 0$, separating variables we get $$\frac{dy}{y} = \frac{xdx}{x^2+1}$$	0.25
		Integrate the above equation $$\int \frac{dy}{y} = \int \frac{xdx}{x^2+1},$$	0.25
		$$\int \frac{dy}{y} = \frac{1}{2} \int \frac{d(x^2+1)}{x^2+1}, \text{ so}$$	0.25
		$$ln\|y\| = \frac{1}{2} ln(x^2+1) + ln\, D, \text{ for } D > 0.$$	0.25 0.25
		Therefore $y = C\sqrt{x^2+1}$, for $C \neq 0$. (2)	0.25
		Sumarizing (1) and (2) we conclude that every solution of the differential equation is of the form $y = C\sqrt{x^2+1}$, for $C \in \mathbb{R}$.	0.25

Exercise		Bref solution of the Final Exam, Subject No. 02 Course Syllabus Math 17B- NH05005 Duration 90 minutes- Date June , 2015	Point
1 **(3 p.)**	a)	Use the substitution $t = \sqrt{x}$, hence $dx = 2tdt$	0.25
		Change the limits of integration $x = 0$, so $u = 0$; $x = \left(\frac{\pi}{2}\right)^2$, so $u = \frac{\pi}{2}$;	0.25
		The initial integral becomes $2\int_0^{\frac{\pi}{2}} t\, cos\, t\, dt$	0.25
		By IBP we get $2\int_0^{\frac{\pi}{2}} t\, cos\, t\, dt = 2\int_0^{\frac{\pi}{2}} t\, d(sin\, t)\, dt$	0.25
		$= 2\left[[t\, sin\, t]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} sin\, t\, dt\right]$	0.25
		. $2\left[[t\, sin\, t]_0^{\frac{\pi}{2}} + [cos\, t]_0^{\frac{\pi}{2}}\right] = \pi - 2.$	0.25
	b)	Making the long division $\frac{x^4 + 4x^2 + 2x - 1}{x^3 + x^2} = x - 1 + \frac{5x^2 + 2x - 1}{x^3 + x^2},$	0.25
		and partial fraction: $\frac{x^4+4x^2+2x-1}{x^3+x^2} = x - 1 + \frac{3}{x}$	0.25
		$-\frac{1}{x^2}$	0.25
		$+\frac{2}{x+1}$	0.25
		Hence the integral is equal to $\frac{x^2}{2} - x + 3ln\lvert x\rvert + \frac{1}{x} + 2ln\lvert x+1\rvert + C$, for $C \in \mathbb{R}$.	0.25 0.25
2 **(3 p.)**	a)	The width of a subinterval is $\Delta_k = \frac{2-0}{n} = \frac{2}{n}$,	0.25
		hence the partition is given by the subintevals $[x_k, x_{k+1}]$, where $x_0 = 0, x_1 = 1.\frac{2}{n}, x_2 = 2.\frac{2}{n}, \ldots, x_k = k.\frac{2}{n}, \ldots, x_n = n.\frac{2}{n} = 2.$	0.25
	b)	Choosing $c_k = x_k = k.\frac{2}{n}, k = 1, \ldots, n,$	0.25
		then the Riemann sum associated is $\sigma_n = \frac{2}{n}\sum_{k=1}^{n}(x_k^2 + 2x_k) = \frac{2}{n}\left[\sum_{k=1}^{n}\left(k.\frac{2}{n}\right)^2 + 2\sum_{k=1}^{n} k.\frac{2}{n}\right]$	0.25
		$= \frac{2}{n}\left[\frac{4}{n^2}\sum_{k=1}^{n} k^2 + \frac{4}{n}\sum_{k=1}^{n} k\right]$	0.25
		$= \frac{2}{n}\left[\frac{4}{n^2}\frac{n(n+1)(2n+1)}{6} + \frac{4}{n}\frac{n(n+1)}{2}\right]$	0.25 0.25

		$= \frac{4(5n^2+6n+1)}{3n^3}$.	0.25
	c)	$\lim_{n\to\infty} \sigma_n = \frac{20}{3}$,	0.25
		hence the integral is $\frac{20}{3}$.	0.25
	d)	Draw the graph of the function $f(x) = x^2 + x$ on $[0,2]$.	0.25
		A geometric meaning of the definite integral: its value, $\frac{20}{3}$ is the area of the region enclosed by the graph of f, the x-axis , and the vertical line $x = 2$.	0.25
3 (2 p.)		For every $x \geq 1$ we have $0 < x^2 + 1 \leq x^2 + x^2 = 2x^2$, and so $$\frac{1}{\sqrt[3]{x^3+1}} \geq \frac{1}{\sqrt[3]{2x^2}} = \frac{1}{\sqrt[3]{2}} x^{-2/3} > 0.$$	0.25 0.25
		On the other hand the impoper integral $\int_1^{+\infty} \frac{1}{\sqrt[3]{2}} x^{-2/3} = \frac{1}{\sqrt[3]{2}} \left[\frac{x^{1/3}}{1/3}\right]_1^{\infty} = +\infty$ is divergent,	1
		therefore the given integral is also divergent by comparison.	0.5
4 (2 p.)		It is clear that $y \equiv 0$ is a particular solution of the eqution. (1)	0.25
		If $y \neq 0$, separating variables we get $$\frac{dy}{y^2} = \sin x dx$$	0.25
		Integrate the above equation $$\int \frac{dy}{y^2} = \int \sin x dx,$$	0.25 0.25
		$\frac{1}{y} = \cos x + C$ for $C \in \mathbb{R}$, so	0.25 0.25
		Therefore $y = \frac{1}{\cos x + C}$, for $C \in \mathbb{R}$. (2)	0.25
		Conclusion : the solutions of the equation are given by (1) and (2).	0.25