Vietnam National University of Agriculture
Faculty of Information Technology
Department of Mathematics

The school year 2015-2016

Final Exam, Subject No. 01
Course Syllabus math 17A- NH05004
Duration 75 minutes

Unauthorized materials

Date December 24, 2015

Exercise 1 (3 points): let the function $f(x,y)=\sqrt{xy+x+1}$.

1) **(1 p.)** Find the partial derivatives of the function.
2) **(1 p.)** Find the linearization of the above function at the point $(1,2)$.
3) **(1 p.)** Using the question 2) to give an approximation of $f(1.02,1.99)$.

Exercise 2 (3 points): let $A=\begin{bmatrix}1 & -2\\ 2 & 6\end{bmatrix}$, $B=\begin{bmatrix}2 & 1\\ 1 & -1\end{bmatrix}$.

1) **(1.5 p.)** Find the inverse matrix (if it exists) of A.
2) **(1.5 p.)** Solve the equation $XA=B$.

Exercise 3 (4 points):

1) **(1 p.)** Find the polar coordinate representation of the vector $x=\begin{bmatrix}-\sqrt{3}\\ 1\end{bmatrix}$.
2) **(1 p.)** Normalize the vector $x=[2,-1,1]'$.
3) **(1 p.)** Find the dot product of $u=[-1,2,2]'$ and $v=[1,3,1]'$.
4) **(1 p.)** Use a rotation matrix to rotate the vector $[3,4]'$ counterclockwise by the angle $\pi/3$.

Written by Quang Sang Phan

Reviewed by

Vietnam National University of Agriculture
Faculty of Information Technology
Department of Mathematics

The school year 2015-2016

Final Exam, Subject No. 02
Course Syllabus math 17A- NH05004
Duration 75 minutes

Unauthorized materials

Date December 24, 2015

Exercise 1 (3 points): let the function $f(x,y)=\sqrt{xy+y+1}$.

1) **(1 p.)** Find the partial derivatives of the function.
2) **(1 p.)** Find the linearization of the above function at the point $(2,1)$.
3) **(1 p.)** Using the question 2) to give an approximation of $f(1.99,1.02)$.

Exercise 2 (3 points): let $A=\begin{bmatrix} 6 & -2 \\ 2 & 1 \end{bmatrix}$, $B=\begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix}$.

1) **(1.5 p.)** Find the inverse matrix (if it exists) of A.
2) **(1.5 p.)** Solve the equation $AX = B$.

Exercise 3 (4 points):

1) **(1 p.)** Find the polar coordinate representation of the vector $x=\begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}$.
2) **(1 p.)** Normalize the vector $x = [1,-2,4]'$.
3) **(1 p.)** Find the dot product of $u = [1,2,1]'$ and $v = [-1,3,-4]'$.
4) **(1 p.)** Use a rotation matrix to rotate the vector $[2,3]'$ counterclockwise by the angle $\pi/6$.

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Exercise		Bref solution of the Final Exam, Subject No. 01 Course Syllabus Math 17A- Duration 75 minutes- Date Dec. 24, 2015	Point
EX 1 (3 p.)	**1)**	$f_x(x,y)=\dfrac{y+1}{2\sqrt{xy+x+1}}$	0.5
		$f_y(x,y)=\dfrac{x}{2\sqrt{xy+x+1}}$	0.5
	2)	Compute $f(1,2)=2,\ f_x(1,2)=\dfrac{3}{4},\ f_y(1,2)=\dfrac{1}{4}$	0.5
		Then the linearization of f at the point $(1,2)$ is $z(x,y)=2+\dfrac{3}{4}(\mathrm{x}-1)+\dfrac{1}{4}(\mathrm{y}-2)=\dfrac{3}{4}\mathrm{x}+\dfrac{1}{4}y+\dfrac{3}{4}$	0.5
	3)	$f(1.02,1.99)\approx\dfrac{3}{4}1{,}02+\dfrac{1}{4}1.99+\dfrac{3}{4}=2.0125$	1.0
EX 2 (3 p.)	**1)**	$\det(\mathrm{A})=10\neq 0$ so A is inversible.	0.5
		$A^{-1}=\dfrac{1}{10}\begin{bmatrix}6 & 2\\ -2 & 1\end{bmatrix}=\begin{bmatrix}3/5 & 1/5\\ -1/5 & 1/10\end{bmatrix},$	1.0
	2)	Due to the fact that A is inversible the equation $XA=B$ is equivalent to $X=BA^{-1}$.	0.5
		$X=\dfrac{1}{10}\begin{bmatrix}2 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}6 & 2\\ -2 & 1\end{bmatrix}$	0.5
		$=\dfrac{1}{10}\begin{bmatrix}10 & 5\\ 8 & 1\end{bmatrix}=\begin{bmatrix}1 & 1/2\\ 4/5 & 1/10\end{bmatrix}$	0.5
EX 3 (4 p.)	**1)**	$r=2,\ \tan\theta=-\dfrac{1}{\sqrt{3}}$ so $\theta=\dfrac{5\pi}{6}$. **(Remark that we need choose $\theta\in[0,\pi)$)**	0.75
		Hence $(2,\dfrac{5\pi}{6})$ is the polar coordinates of the point.	0.25
	2)	$\lvert x\rvert=\sqrt{6}$ and the normalization of the vector is $\left[\dfrac{2}{\sqrt{6}},\dfrac{-1}{\sqrt{6}},\dfrac{1}{\sqrt{6}}\right]$,	0.5 0.5
	3)	$uv=-1+6+2=7$	1.0
	4)	The matrix of the rotation by the angle $\theta=\dfrac{\pi}{3}$ is	
		$R_\theta=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}=\begin{bmatrix}\dfrac{1}{2} & -\dfrac{\sqrt{3}}{2}\\ \dfrac{\sqrt{3}}{2} & \dfrac{1}{2}\end{bmatrix},\quad R_\theta\begin{bmatrix}3\\ 4\end{bmatrix}=\begin{bmatrix}\dfrac{3-4\sqrt{3}}{2}\\ \dfrac{4+3\sqrt{3}}{2}\end{bmatrix}$	0.5 0.5

Exercise		Bref solution of the Final Exam, Subject No. 02 Course Syllabus Math 17A-Duration 75 minutes- Date Dec. 24, 2015	Point
EX 1 (3 p.)	**1)**	$f_x(x,y)=\dfrac{y}{2\sqrt{xy+y+1}}$	0.5
		$f_y(x,y)=\dfrac{x+1}{2\sqrt{xy+y+1}}$	0.5
	2)	Compute $f(2,1)=2$, $f_x(2,1)=\dfrac{1}{4}$, $f_y(2,1)=\dfrac{3}{4}$	0.5
		Then the linearization of f at the point $(1,2)$ is $z(x,y)=2+\dfrac{1}{4}(\mathrm{x}-2)+\dfrac{3}{4}(\mathrm{y}-1)=\dfrac{1}{4}\mathrm{x}+\dfrac{3}{4}y+\dfrac{3}{4}$	0.5
	3)	$f(1.99,1.02)\approx\dfrac{1}{4}1.99+\dfrac{3}{4}1{,}02+\dfrac{3}{4}=2.0125$	1.0
EX 2 (3 p.)	**1)**	$\det(\mathrm{A})=10\neq 0$ so A is inversible.	0.5
		$A^{-1}=\dfrac{1}{10}\begin{bmatrix}1 & 2\\ -2 & 6\end{bmatrix}=\begin{bmatrix}1/10 & 1/5\\ -1/5 & 3/5\end{bmatrix}$,	1.0
	2)	Due to the fact that A is inversible the equation $AX=B$ is equivalent to $X=A^{-1}B$.	0.5
		$X=\dfrac{1}{10}\begin{bmatrix}1 & 2\\ -2 & 6\end{bmatrix}\begin{bmatrix}-1 & 3\\ 1 & 2\end{bmatrix}$ $=\dfrac{1}{10}\begin{bmatrix}1 & 7\\ 8 & 6\end{bmatrix}=\begin{bmatrix}1/10 & 7/10\\ 4/5 & 3/5\end{bmatrix}$	0.5 0.5
EX 3 (4 p.)	**1)**	$r=2$, $\tan\theta=-\sqrt{3}$ so $\theta=\dfrac{2\pi}{3}$. **(Remark that we need choose $\theta\in[0,\pi)$)** Hence $(2,\dfrac{2\pi}{3})$ is the polar coordinates of the point.	0.75 0.25
	2)	$\lvert x\rvert=\sqrt{21}$ and the normalization of the vector is $\left[\dfrac{1}{\sqrt{21}},\dfrac{-2}{\sqrt{21}},\dfrac{4}{\sqrt{21}}\right]'$	0.5 0.5
	3)	$uv=-1+6-4=1$	1.0
	4)	The matrix of the rotation by the angle $\theta=\dfrac{\pi}{6}$ is $R_\theta=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}=\begin{bmatrix}\dfrac{\sqrt{3}}{2} & -\dfrac{1}{2}\\ \dfrac{1}{2} & \dfrac{\sqrt{3}}{2}\end{bmatrix}$, $R_\theta\begin{bmatrix}2\\ 3\end{bmatrix}=\begin{bmatrix}\dfrac{-3+2\sqrt{3}}{2}\\ \dfrac{2+3\sqrt{3}}{2}\end{bmatrix}$	0.5 0.5