

CODE: No.1 - FE1718

FACULTY OF INFORMATION  
TECHNOLOGY  
DEPARTMENT OF MATHEMATICS

SOCIAL REPUBLIC OF VIETNAM  
Independence - Freedom - Happiness

FINAL EXAM  
Times: 75 minutes  
THE01002. CALCULUS 2

Description of the test : this test includes 4 exercises. Points are distributed as follows:

Question	1	2	3	4
Points	1.5	3.5	1.5	3.5

**Exercise 1.** 1.5 pts Solve the differential equation:

$$\frac{dy}{dx} = \frac{y}{x+1}, \text{ where } y_0 = 1 \text{ when } x_0 = 1.$$

**Exercise 2.** 3.5 pts Suppose that the size  $N(t)$  of a population at time  $t$  according to the equation

$$\frac{dN}{dt} = \frac{1}{200}N^2, \text{ with } N(0) = 20.$$

- a) 1.5 pts Solve the equation and find the size of the population at time  $t = 5$ .
- b) 1.0 pt Graph  $N(t)$  as a function of  $t$  for  $0 \leq t \leq 10$ . What happens as  $t \rightarrow 10$ ? Explain in words what this means.
- c) 1.0 pt Find the average size of the population between  $t = 0$  and  $t = 5$ .

**Exercise 3.** 1.5 pts Let  $N(t)$  denote the size of a population at time  $t$ . Assume that the population evolves according to the logistic equation. Suppose that then intrinsic growth rate is 4 and the carrying capacity is 50.

- a) 1.0 pt Find a differential equation that describes the growth of this population.
- b) 0.5 pt Without solving the differential equation in a), sketch solution curve of  $N(t)$  when  $N(0) = 15$ .

**Exercise 4.** 3.5 pts Suppose that a fish population evolves according to the logistic equation and that a fixed number of fish per unit time are removed. That is,

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - H.$$

Assume that  $r = 2$ ,  $K = 100$ , and  $H = 48$

- a) 1.5 pt Find the equilibria of the equation.
- b) 2.0 pts Analyze their stability, using both: the graphical approach, and the analytical approach.

LECTURER  
Quang Sang PHAN

REVIEWED BY

CODE: No.2 - FE1718

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FINAL EXAM  
Times: 75 minutes  
THE01002. CALCULUS 2

Description of the test : this test includes 4 exercises. Points are distributed as follows:

Question	1	2	3	4
Points	1.5	3.5	1.5	3.5

**Exercise 1.** [1.5 pts] Solve the differential equation:

$$\frac{dy}{dx} = \frac{y+1}{x}, \text{ where } y_0 = 2 \text{ when } x_0 = 1.$$

**Exercise 2.** [3.5 pts] Suppose that the size  $N(t)$  of a population at time  $t$  according to the equation

$$\frac{dN}{dt} = \frac{1}{100}N^2, \text{ with } N(0) = 10.$$

- a) [1.5 pts] Solve the equation and find the size of the population at time  $t = 5$ .
- b) [1.0 pt] Graph  $N(t)$  as a function of  $t$  for  $0 \leq t \leq 10$ . What happens as  $t \rightarrow 10$ ? Explain in words what this means.
- c) [1.0 pt] Find the average size of the population between  $t = 0$  and  $t = 5$ .

**Exercise 3.** [1.5 pts] Let  $N(t)$  denote the size of a population at time  $t$ . Assume that the population evolves according to the logistic equation. Suppose that then intrinsic growth rate is 3 and the carrying capacity is 40.

- a) [1.0 pt] Find a differential equation that describes the growth of this population.
- b) [0.5 pt] Without solving the differential equation in a), sketch solution curve of  $N(t)$  when  $N(0) = 10$ .

**Exercise 4.** [3.5 pts] Suppose that a fish population evolves according to the logistic equation and that a fixed number of fish per unit time are removed. That is,

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - H.$$

Assume that  $r = 2$ ,  $K = 200$ , and  $H = 96$

- a) [1.5 pt] Find the equilibria of the equation.
- b) [2.0 pts] Analyze their stability, using both: the graphical approach, and the analytical approach.

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Quang Sang PHAN

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