CODE 1: FIN-TH1-1819

FACULTY OF INFORMATION TECHNOLOGY
DEPARTMENT OF MATHEMATICS
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Independence - Freedom - Happiness
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FINAL EXAM CALCULUS 1 (THE01001)

Times: 75 minutes

Description of the test : this test includes 3 exercises with 7 questions. Points are distributed as follows:

Question	1	2	3	4	5	6	7
Points	2.0	1.0	1.5	1.0	1.0	2.0	1.5

Exercise 1. $\boxed{2.0 \text{ pt}}$ Let the following matrixes

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 2 & -1 \\ -1 & 3 \\ 2 & 0 \end{bmatrix}.$$

Find $A^t + 2B$ and AB.

Exercise 2.

a) $\boxed{1.0 \text{ pt}}$ Find the image of the vector $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ by the linear map associated with the matrix

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}.$$

b) $\boxed{1.5 \text{ pt}}$ Use a rotation matrix to rotate the vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ counterclockwise by the angle $\dfrac{\pi}{3}$.

Exercise 3. Let the function

$$f(x, y) = \sqrt{9 - x^2 - y^2}.$$

a) $\boxed{1.0 \text{ pt}}$ Give a geometrical representation for the domain of f and find the range of f.

b) $\boxed{1.0 \text{ pt}}$ Determine level curves of the function.

c) $\boxed{2.0 \text{ pt}}$ Find the partial derivatives of the function and then compute

$$x f_x + y f_y.$$

d) $\boxed{1.5 \text{ pt}}$ Find the standard linear approximation of the function at the point $(1, -2)$ and then use it to find an approximation for $f(1.01; -2.02)$.

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SOLUTION FOR THE FINAL EXAM CODE 1: FIN-TH1-1819

Exercise 1. $\boxed{2.0\text{ pt}}$ $A^t + 2B = \begin{bmatrix} 1 & 2 \\ -1 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -2 & 6 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -3 & 3 \\ 6 & 1 \end{bmatrix}$ and $AB = \begin{bmatrix} 7 & -4 \\ 9 & -11 \end{bmatrix}$.

Exercise 2. $\boxed{2.5\text{ pt}}$

a) $\boxed{1.0\text{ pt}}$ The image of the vector $x = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ by the linear map is

$$Ax = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}.$$

b) $\boxed{1.5\text{ pt}}$ The rotation matrix is $R_{\frac{\pi}{3}} = \begin{bmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$. Then the rotation of the vector $x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ is the vector

$$R_{\frac{\pi}{3}} x = \begin{bmatrix} \dfrac{3 - 4\sqrt{3}}{2} \\ \dfrac{4 + 3\sqrt{3}}{2} \end{bmatrix}$$

Exercise 3. The function $f(x, y) = \sqrt{9 - x^2 - y^2}$.

a) $\boxed{1.0\text{ pt}}$ $D_f = \{(x, y) \in \mathbb{R}^2 |\ x^2 + y^2 \leq 9\}$, so the domain of f is inside the circle of center $(0, 0)$ and radius 3. The range of f is $R(f) = [0; 3]$

b) $\boxed{1.0\text{ pt}}$ Let $c \in R(f)$, $0 \leq c \leq 3$, the level curves of f are determined by the equations

$$\sqrt{9 - x^2 - y^2} = c \Leftrightarrow x^2 + y^2 = 9 - c^2.$$

The level curves are a family of circles of center $(0, 0)$ and radius $\sqrt{9 - c^2}$.

Draw some particular level curves...

c) $\boxed{2.0\text{ pt}}$ The partial derivatives

$$f_x = \frac{-x}{\sqrt{9 - x^2 - y^2}}, \quad f_y = \frac{-y}{\sqrt{9 - x^2 - y^2}}.$$

Then

$$x f_x + y f_y = -\frac{x^2 + y^2}{\sqrt{9 - x^2 - y^2}}.$$

d) $\boxed{1.5\text{ pt}}$ $f(1, -2) = 2,\ f_x(1, -2) = -\frac{1}{2},\ f_y(1, -2) = 1.$

The standard linear approximation of the function at the point $(1, -2)$ is

$$L(x, y) = 2 - \frac{1}{2}(x - 1) + (y + 2) = -\frac{1}{2}x + y + \frac{9}{2}.$$

$$f(1.01; -2.02) \approx L(1.01; -2.02) = -\frac{1}{2} \times 1.01 - 2.02 + \frac{9}{2} = 2.475.$$