CODE: MID-TH2-1819

FACULTY OF INFORMATION TECHNOLOGY
DEPARTMENT OF MATHEMATICS

SOCIAL REPUBLIC OF VIETNAM
Independence - Freedom - Happiness

MIDTERM EXAM- March 5, 2019

Times: 50 minutes

THE01002. CALCULUS 2

Description of the test : this test includes 5 exercises with 6 questions. Points are distributed as follows:

Question	1	2	3	4	5	6
Points	1.5	2.5	1.5	1.5	1.5	1.5

Exercise 1. 1.5 pt Express the following limit as a definite integral

$$\lim_{\|P\|\to 0} \sum_{k=1}^{n} \frac{2c_k+1}{1+c_k^2} \triangle x_k,$$

where $P = [x_1, \cdots, x_n]$ is a partition of the interval $[1, 2]$; $c_k \in [x_{k-1}, x_k]$, and $\triangle x_k = x_k - x_{k-1}$.

Exercise 2. 2.5 pt Express the integral $\int_0^1 \sqrt{x+1}dx$ as a limit of Riemann sums. Do not evaluate the limit.

Exercise 3. 1.5 pt Find the derivative with respect to x of the function

$$G(x) = \int_1^{x^2+1} \ln(t^2+1)dt.$$

Exercise 4. 3.0 pt Evaluate the following integrals

a) 1.5 pt $\int \frac{x^2+2x}{\sqrt{x}}dx$.

b) 1.5 pt $\int_e^{+\infty} \dfrac{dx}{x\sqrt[3]{\ln^4 x}}$ (if it exists).

Exercise 5. 1.5 pt Suppose that the temperature (measured in degrees Fahrenheit) in a growing chamber varies over a 24-hour period according to

$$T(t) = 68 + \sin(\frac{\pi}{12}t),$$

for $0 \le t \le 24$. Find the average temperature of the chamber.

LECTURER
Quang Sang PHAN

CORRECTION for MIDTERM EXAM- March 5, 2019

Exercise 1. $\boxed{1.5 \text{ pt}}$ Express the following limit as a definite integral

$$\lim_{\|P\|\to 0} \sum_{k=1}^{n} \frac{2c_k+1}{1+c_k^2} \triangle x_k = \int_1^2 \frac{2x+1}{1+x^2} dx.$$

Exercise 2. $\boxed{2.5 \text{ pt}}$ The function is continuous on $[0,1]$ so is integrable on $[0,1]$. We use an equal partition P for $[0,1]$ with

$$x_0 = 0, x_1 = \frac{1}{n}, \cdots, x_k = \frac{k}{n}, \cdots, x_n = 1.$$

This partition satisfies $\|P\| = \frac{1}{n} \to 0$ as $n \to +\infty$ We choose the sampling points as right (or left) endpoints of the subintervals:

$$c_1 = \frac{1}{n}, c_2 = \frac{2}{n}, \cdots, c_k = \frac{k}{n}, \cdots, c_n = 1.$$

Then the integral can express as a limit of the corresponding Riemann sum

$$\int_0^1 \sqrt{x+1} dx = \lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{\frac{k}{n}+1} \frac{1}{n}.$$

Exercise 3. $\boxed{1.5 \text{ pt}}$

$$G'(x) = \ln((x^2+1)^2+1)\ (x^2+1)' = 2x \ln(x^4+2x^2+2).$$

Exercise 4. $\boxed{3.0 \text{ pt}}$ Evaluate the following integrals

a) $\boxed{1.5 \text{ pt}}$ $\int \frac{x^2+2x}{\sqrt{x}} dx = \int (x^{\frac{3}{2}} + 2x^{\frac{1}{2}}) dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + C = \frac{2}{5}x^2\sqrt{x} + \frac{4}{3}x\sqrt{x} + C,\ C \in \mathbb{R}$

b) $\boxed{1.5 \text{ pt}}$ Let $t = \ln x$, then $dt = \frac{dx}{x}$.

When $x = e$ then $t = 1$, and $t \to +\infty$ as $x \to +\infty$.

$$\int_e^{+\infty} \frac{dx}{x\sqrt[3]{\ln^4 x}} = \int_1^{+\infty} \frac{dt}{\sqrt[3]{t^4}} = \int_1^{+\infty} t^{\frac{-4}{3}} = -3t^{\frac{-1}{3}}]_1^{+\infty} = -3 \lim_{t\to+\infty} t^{\frac{-1}{3}} + 3 = 3.$$

Exercise 5. $\boxed{1.5 \text{ pt}}$

$$T_{avg} = \frac{1}{24}\int_0^{24} (68 + \sin\frac{\pi t}{12}) dt = \frac{1}{24}[68t - \frac{12}{\pi}\cos\frac{\pi t}{12}]_0^{24} = \frac{1}{24}[68 \times 24 - \frac{12}{\pi}\cos(2\pi) + \frac{12}{\pi}\cos 0] = 68.$$

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