Chapter 1. Linear equations

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3. Linear system
4. Supply and demand analysis
5. Transposition of formulas
6. National income determination

Revisions of basic rules used in arithmetic and algebra
*Abbreviation: Practice Problem page $\Rightarrow$ PPp

- Section 1.1:
  - Numbers and basic mathematical operations: ppp 21
  - Expressions: ppp 24-25
  - Brackets: ppp 27-29
- Section 1.2:
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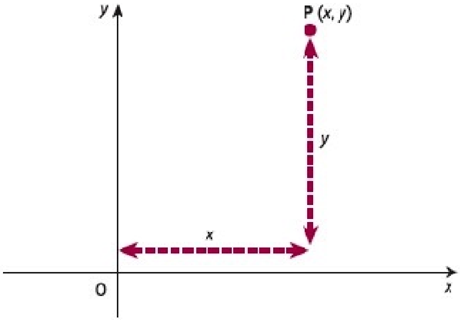
  - Inequalities: ppp 33-34
  - Key terms: page 49

Home work: Ex 1.1 page 33-, Ex 1.2* page 51-

Coordinate system:



Example: plot the points $A(2,3)$, $B(-1,4)$, $C(-3,-1)$, $D(3,-2)$, and $E(5,0)$ on a coordinate plane.

Straight lines and linear equations:

$$ax + by = c, a^2 + b^2 \neq 0,$$

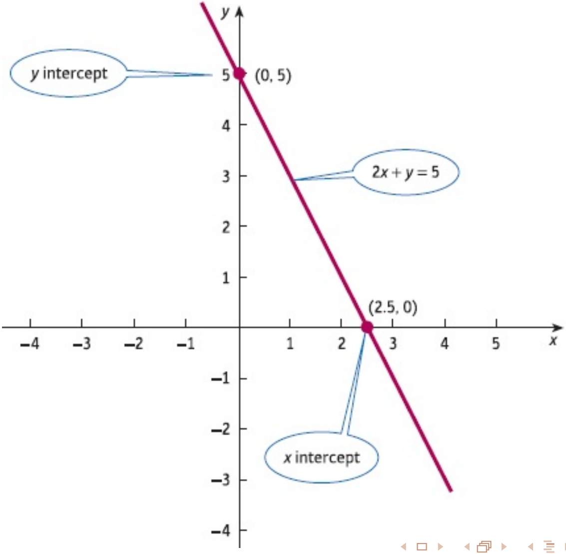
ppp 55, 56

Example: decide which of the following points lie on the line $5x - 2y = 6$: $A(0, -3), B(2, 2)$, $C(-10, -28)$, and $D(4, 8)$.

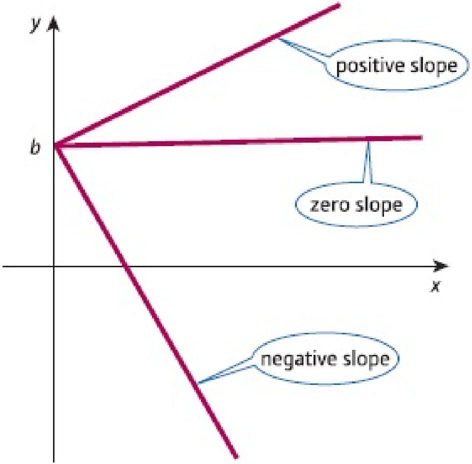
Intercepts and slope: ppp 59,61

Example: draw the graph of the linear equation

$$2x + y = 5$$



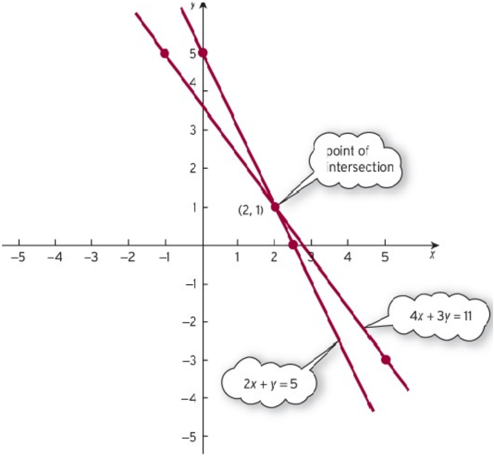
$y = kx + b$, k is the slop of the straight line



Simultaneous linear equations: page 45

Example: find the point of intersection of the two lines

$$2x + y = 5, \; 4x + 3y = 11$$



Example: page 63

HW: Exercise 1.3, page 66, problem 5

Elimination method: ppp 71, 73, 77

Example 1: solve the linear systems

$$3x + 2y = 1$$

$$-2x + y = 2$$

Example 2: solve the linear systems

$$x + 3y - z = 4$$

$$2x + y + 2z = 10$$

$$3x - y + z = 4$$

HW: Exercise 1.4, page 79, problem 1, 4

The quantity demanded, Q, of a good depends on the market price, P, and vice versa. We can write

$$P = g(Q),$$

called the **demand function**.

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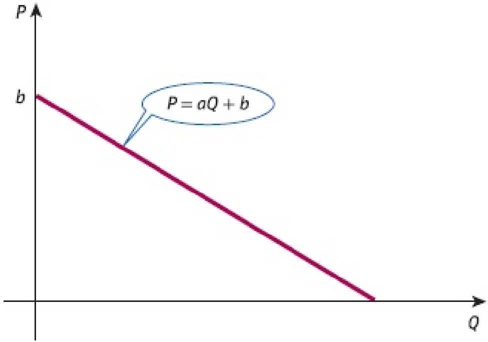
The process of identifying the key features of the real world and making appropriate simplification and assumptions is known as modelling.

A simple model is a linear function:

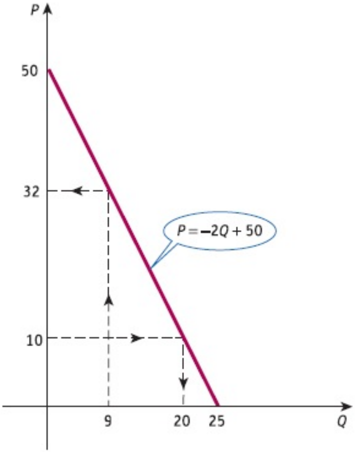
$$P = aQ + b,$$

for some constants a and b.



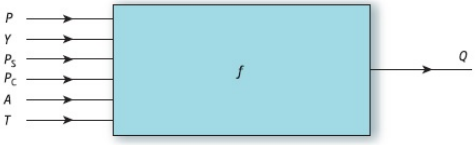
Usually, P is a decreasing function of Q ($a < 0$, $b > 0$).

Example: page 83



*ppp 84

Endogenous and exogenous, page 85



$$Q = f(P, Y, P_S, P_C, A, T) \;\Rightarrow\; P = f(Q, Y, P_S, P_C, A, T)$$

In the previous model we implicitly assumed that the variables Y, P_S, P_C, A, T are fixed. Then Q and P are called endogenous variables. The remaining variables are called exogenous.

Substitutable goods: A pair of goods that are alternatives to each other. As the price of one of them goes up, the demand for the other rises.

Complementary goods: A pair of goods consumed together. As the price of either goes up, the demand for both goods goes down.

Example: The demand Q for a certain good depends on its own price P and the price of an alternative good P_A, according to

$$Q = 30 - 4P + 2P_A.$$

Is the alternative good substitutable or complementary? Give a reason for your answer.

Discussion: (coal and electricity), (moto and car)...

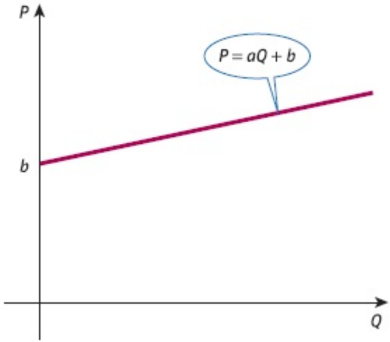
Inferior good: a good whose demand decreases as income increases

Superior good: a good whose demand increases as income increases

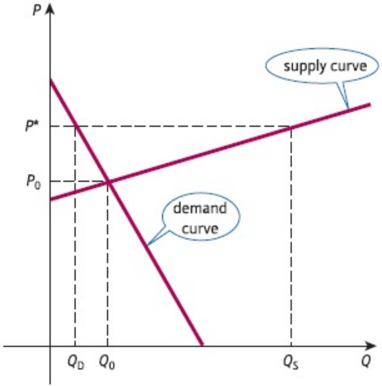
The **supply function** is the relation between the quantity Q of a good that producers plan to bring to the market and the price P of the good.

A linear model for the supply function

$$P = aQ + b, \;\; a > 0, \;\; b > 0$$



Equilibrium price P_0 and quantity Q_0: page 69

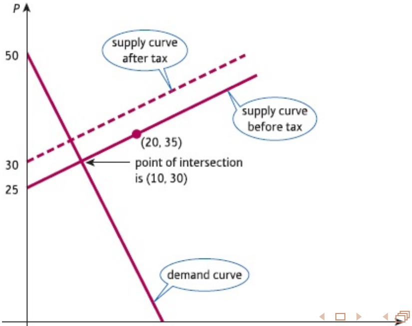


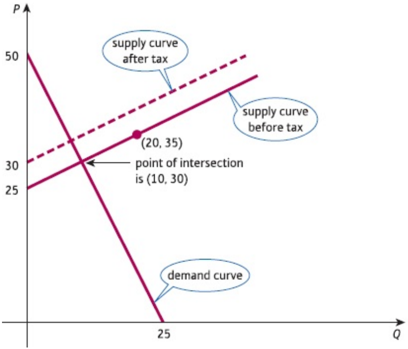
At the point of intersection the market is in equilibrium because the quantity supplied exactly matches the quantity demanded.

Example (page 88): The demand and supply functions of a good are given by

$$P = -2Q_D + 50$$
$$P = \frac{1}{2}Q_S + 25$$

(a) Determine the equilibrium price and quantity.
(b) Determine the effect on the market equilibrium if the government decides to impose a fixed tax of \$5 on each good.





*ppp 90. 92
*Key terms page 93
HW: Exercise 1.5, page 95-, problems 4-5-6

Transpose

$$P = \text{ an expression involving } Q$$

into

$$Q = \text{ an expression involving } P$$

* ppp 98, 101, 104
HW: Exercise 1.6, page 105-, problems 1, 3, 4

National income

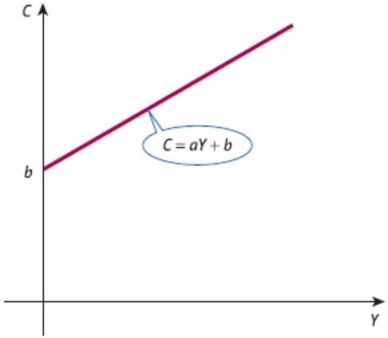
We will set up simple models of the national economy which allows to determine equilibrium levels of income.

We assume that the economy is divided into two sectors, households and firms.

The income of households is used up in consumption and savings. We denote C the **consumption function**, S the **savings**, and Y the income:

$$Y = C + S$$

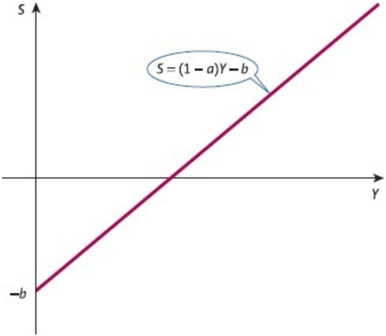
We assume that



Remark $b > 0$, $0 < a < 1$.
The intercept b is known as autonomous consumption (when $Y = 0$).
The slope a is called the marginal propensity to consume (**MPC**).

We have also

$$S = (1 - a)Y - b$$

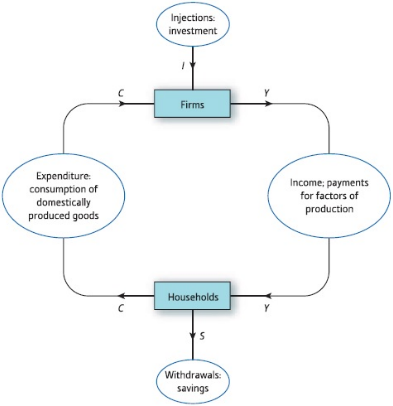


The slope $1 - a$ is called the marginal propensity to save (**MPS**).

The autonomous savings are equal to $S = -b < 0$ (when $Y = 0$).

*Example page 107

The simplest model of the national economy which shows the circular flow of income and expenditure



If the economy is in equilibrium, the flow of income and expenditure balance so that

$$Y = C + I,$$

where I is the **Investment**.

Example, page 110: Find the equilibrium level of income and consumption if the consumption function is

$$C = 0.6Y + 10$$

and planned investment $I = 12$.

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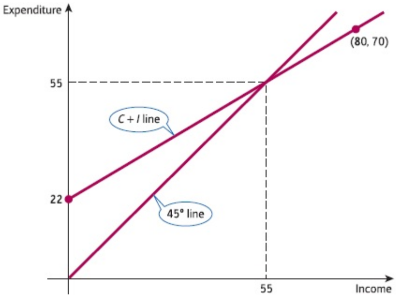
and planned investment $I = 12$.
Solution: since $Y = C + I$, $C = 0.6Y + 10$, $I = 10$ then $Y = 55$, $C = 43$.



*ppp 111

A more complicated model of national economy

For more realistic, we include **government expenditure** G, and **taxation** T in the previous model. Then

$$Y = C + I + G$$

We assume that government expenditure and investment are planed with fixed values:

$$I = I^*, \ G = G^*$$

The income after (less) tax that households have to spend on consumer goods is now

$$Y_d := Y - T,$$

called disposable income.

Hence

$$C = aY_d + b.$$

In practice, T is often given in the forms

$$T = T^*, \text{or } T = tY \text{ for some proportion } t, \text{or } T = tY + T^*$$

Example: Given that $G = 20$, $I = 35$, $C = 0.9Y_d + 70$, $T = 0.2Y + 25$. Calculate the equilibrium level of national income.

Hence

$$C = aY_d + b.$$

In practice, T is often given in the forms

$$T = T^*, \text{or } T = tY \text{ for some proportion } t, \text{or } T = tY + T^*$$

Example: Given that $G = 20$, $I = 35$, $C = 0.9Y_d + 70$, $T = 0.2Y + 25$. Calculate the equilibrium level of national income.

Solution: well remember that from theory

$$Y = C + I + G$$

$$Y_d = Y - T$$

*ppp 113

IS schedule, page 113

$$Y = C + I$$

$$C = aY + b$$

Now we assume that planned Investment I depends on **rate of interest** r:

$$I = cr + d,$$

where $c < 0$, and $d > 0$.
We have a relationship between national income Y and interest rate r:

$$(1 - a)Y = cr + b + d, \text{ or}$$

$$(1 - a)Y - cr = b + d,$$

called the IS schedule. (Investment-Saving schedule, $I = S$)

LM schedule, page 114

The money market is said to be in equilibrium when the **money supply** M_S, matches the **money demand** M_D

$$M_S = M_D.$$

LM schedule, page 114

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How we can determine M_S and M_D.

LM schedule, page 114

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$$M_S = M_D.$$

How we can determine M_S and M_D.

Suppose that M_S can be planed

$$M_S = M_S^*,$$

for some fixed value M_S^*, and

$$M_D = L_1 + L_2,$$

where L_1 denotes the aggregate transaction-precautionary demand, L_2 denotes the speculative demand for money.

We assume that

$$L_1 = k_1 Y, \;\; k_1 > 0,$$

$$L_2 = k_2 r + k_3, \;\; k_2 < 0, \;\; k_3 > 0$$

So the total money demand is

$$M_D = k_1 Y + k_2 r + k_3$$

If the money market is in equilibrium then

$$M_S = M_D \Leftrightarrow M_S^* = k_1 Y + k_2 r + k_3$$

This equation, relating national income Y and interest rate r w.r.t. the money market is called the LM schedule.

Example: page 115, ppp 117

Key term: page 118

*HW: Exercise 1.7, page 118-119

THANK YOU
FOR YOUR ATTENTION!