Chapter 2. Non-linear equations

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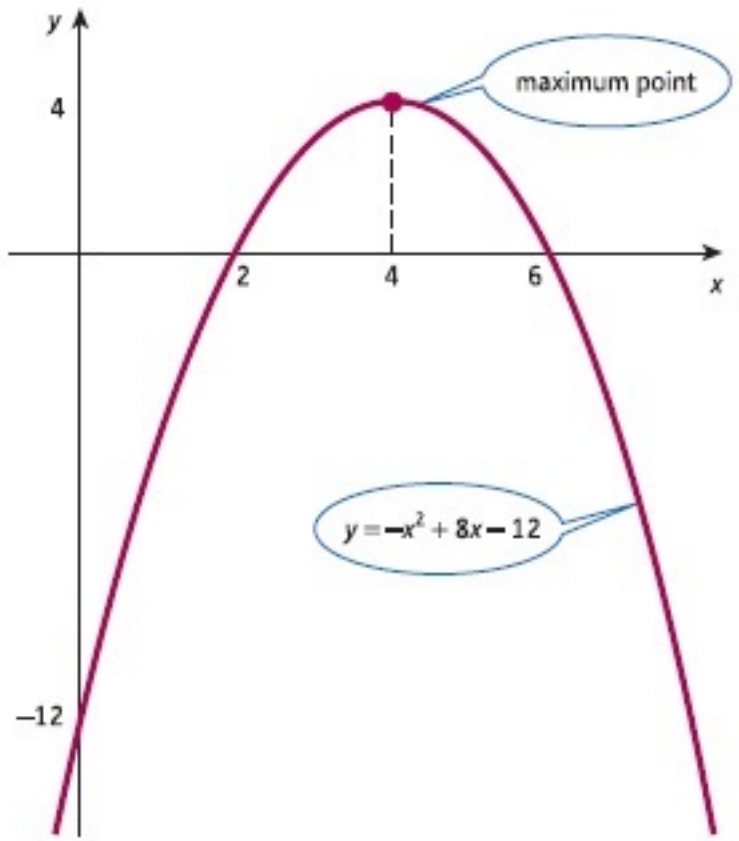
Content

Review:

- Solve a quadratic equation
- Sketch the graph of a quadratic function
- Solve quadratic inequalities

Example: consider the quadratic function

$$f(x) = -x^2 + 8x - 12$$



Example: given the supply and demand functions

$$P = Q_S^2 + 14Q_S + 22$$

$$P = -Q_D^2 - 10Q_D + 150.$$

Calculate the equilibrium price and quantity.

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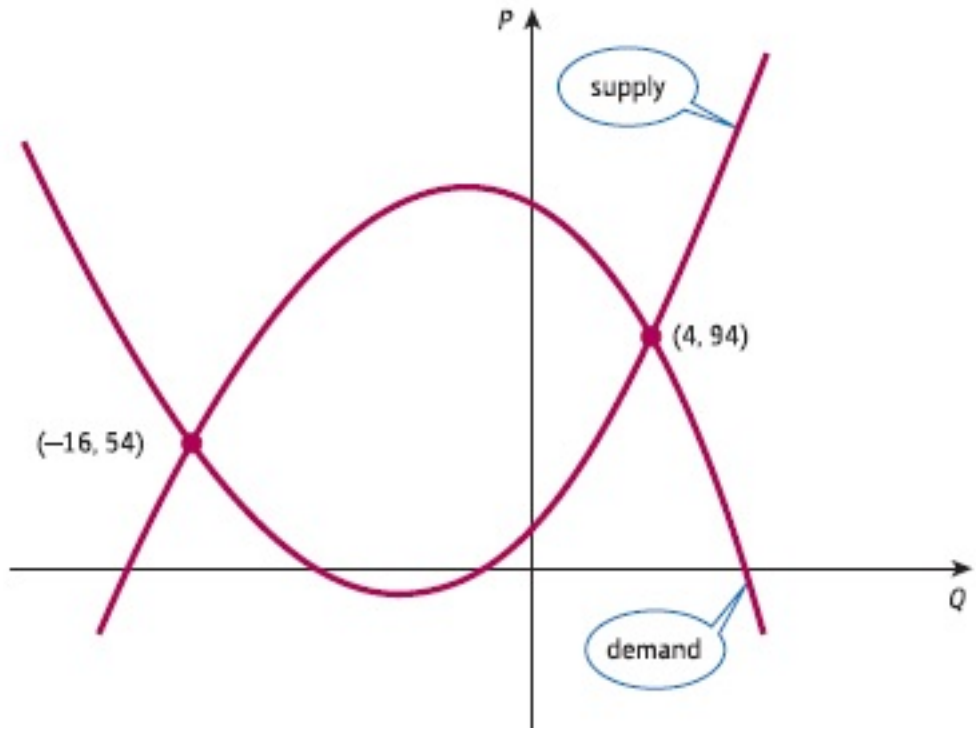

*ppp 138, 139

HW: Exercise 2.1, page 143-, problems 4, 5, 8, 10

Key terms page 141

We donete total revenue TR, and total cost TC.
The profit function, denoted by π, then is

$$\pi = TR - TC.$$

The total revenue received from the sale of Q goods at price P is given by

$$TR = PQ$$

Example, p. 145: Given the demand function $P = 100 - 2Q$. Express TR as a function of Q and hence sketch its graph.

1. For what values of Q is TR zero?
2. What is the maximum value of TR?

*ppp 146

Discussion page 147 about the components of the total cost:
- FC: fixed cost
- TVC: the total variable cost in producing Q goods is

$$TVC = (VC)Q,$$

where VC is the variable cost per unit of output.
Then the total cost TC is

$$TC = FC + TVC = FC + (VC)Q$$

The average cost function AC is defined by the total cost per unit of output

$$AC = \frac{TC}{Q} = \frac{FC}{Q} + VC$$

Example: given that the fixed costs are 1000 and that variable costs are 4 per unit, express the total and average costs as functions of Q. Hence sketch their graphs.

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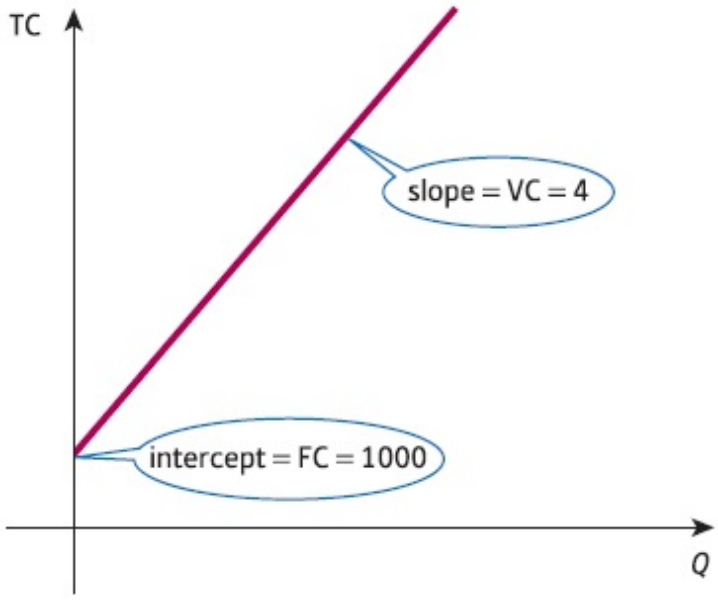
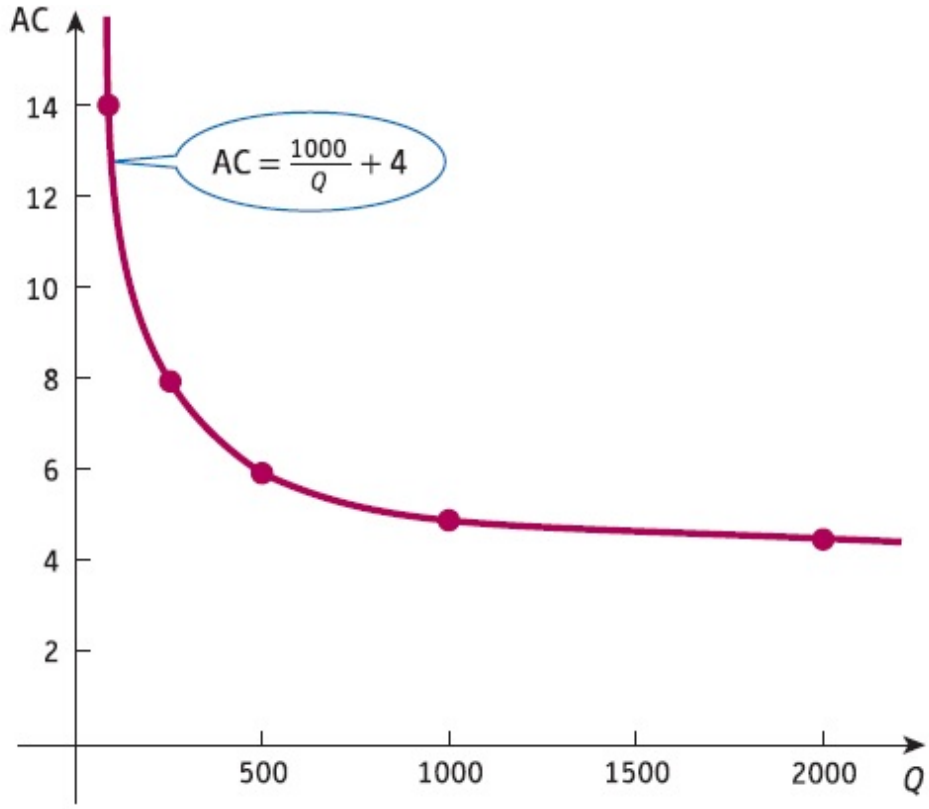


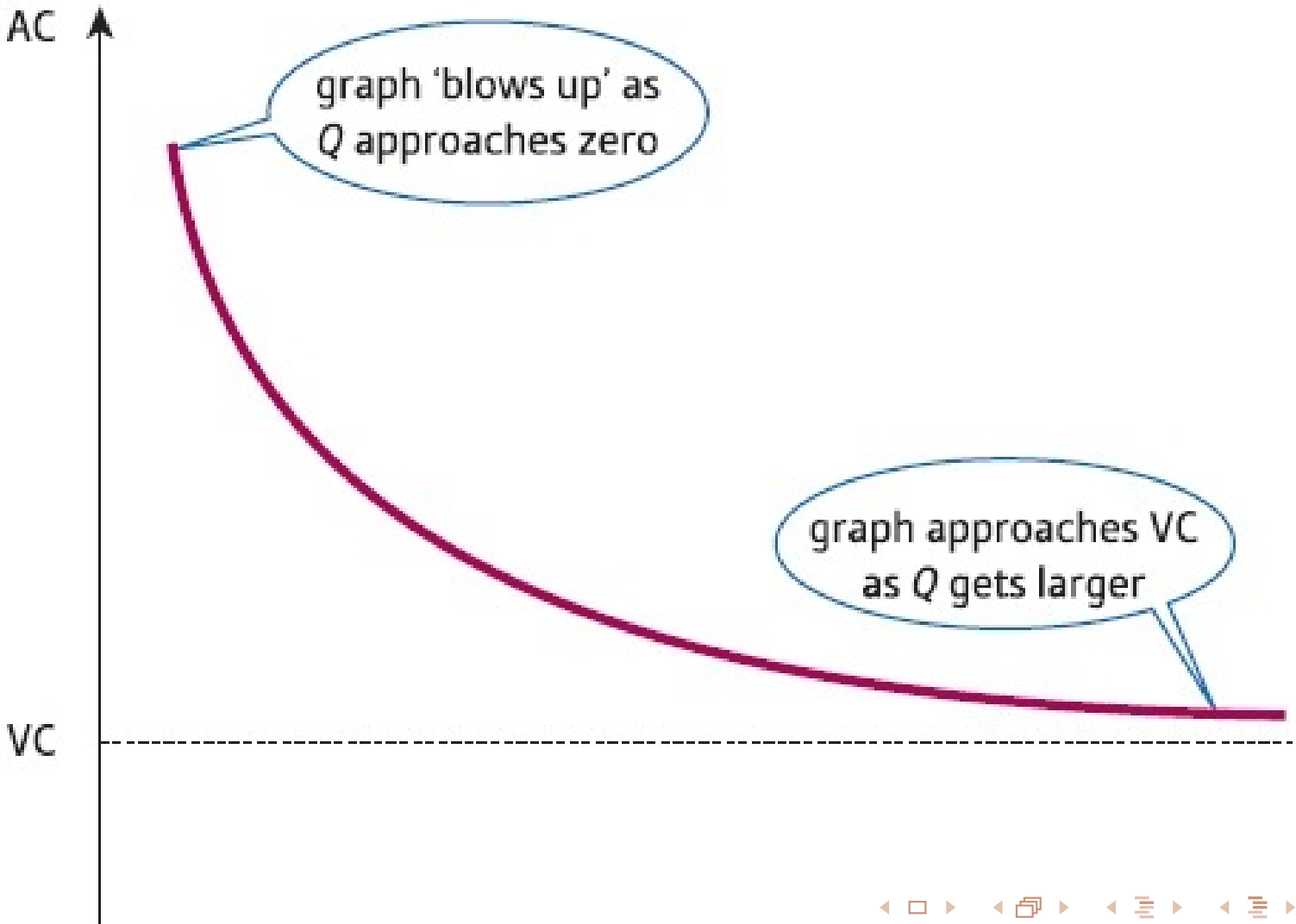


* ppp 149

Discussion (p. 149) on Average cost

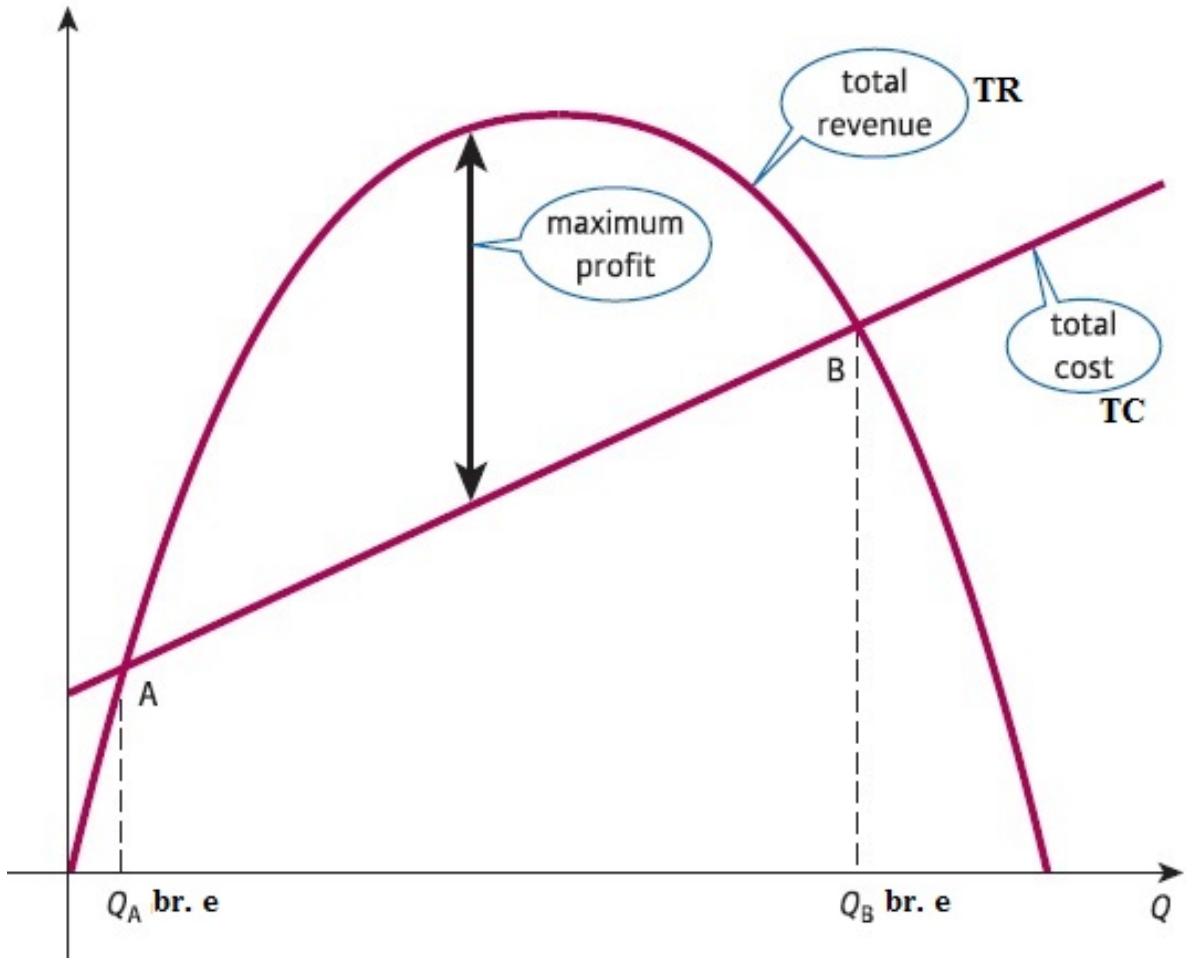
$$AC = \frac{TC}{Q} = \frac{FC}{Q} + VC$$

as either Q is small or large.



Discussion (p. 149-150) on the profit: break even, maximum profit

$$\pi = TR - TC$$



(assuming that the quantity of qoods sold equals to the quantity of goods produced)

Example: page 151

If fixed costs are 4, variable costs per unit are 1 and the demand function is

$$P = 10 - 2Q$$

obtain an expression for π in terms of Q and hence sketch a graph of π against Q.

(a) For what values of Q does the firm break even?

(b) What is the maximum profit?

* ppp 152

* Key terms page 153

*HW: Exercise 2.2, page 153-154

Review : Index notation and Rules of indices

$$b^n,\ b^{-n},\ b^{\frac{1}{n}},\ b^{\frac{m}{n}}$$

Production function, page 164

The output Q of any production process depends on a variety of inputs, known as factors of production.
For simplicity we restrict our attention to capital and labour:

$$Q = f(K, L),$$

where K denotes all man-made aids to production such as buildings, tools and plant machinery, and L denotes all paid work in the production process.
Q is called a **production function**.
Example: a production function is given by

$$Q = 2K^{1/2}L^{3/2}$$

Logarithms

For $0 < b \neq 1$,

$$x = b^y \iff y = \log_b x$$

In particular, we call logarithms to base e natural logarithms

$$x = e^y \iff y = \log_e x = \ln x$$

*HW: problem 13, page 175

Graph of exponential and logarithm functions

Example: page 180, 183, 185

* ppp 182, 184, 187

*HW: Exercise 2.4, page 187-, problems 1, 5,

HW: Exercise 2.4, page 188-, problems 1, 3, 5

THANK YOU VERY MUCH!