CODE 1: MID-TH1-1920

FACULTY OF INFORMATION TECHNOLOGY
DEPARTMENT OF MATHEMATICS
School year 2019-2020

SOCIAL REPUBLIC OF VIETNAM
Independence - Freedom - Happiness
October 10,2019

CALCULUS 1 (THE01001) - MIDTERM EXAM

Times: 55 minutes

Description of the test: this is a closed-book exam, any support material is not permitted; this test includes 3 problems with 9 questions. Points are distributed as follows:

Question	1	2	3	4	5	6	7	8	9
Points	1.0	1.0	1.0	1.5	1.0	1.0	1.0	1.0	1.5

Problem 1. [2.0 pt] Let the function

$$f(x) = \begin{cases} \frac{1}{x} & \text{for} \quad x \geq 1 \\ 2x + c & \text{if} \quad x < 1 \end{cases}$$

a) [1.0 pt] Graph f when $c = 0$, and determine whether f is continuous for this choice of c.

b) [1.0 pt] How much you choose c so that f is continuous for all $x \in (-\infty, +\infty)$.

Problem 2. [2.5 pt] Assume that the size of a population at time t is given by

$$N(t) = \frac{200t}{3+t}, \ t \geq 0.$$

a) [1.0 pt] Determine the limiting population size, that is the size of population as $t \to +\infty$.

b) [1.5 pt] Find the growth rate and then the per capital growth rate of the population at time $t = 1$.

Problem 3. [5.5 pt] A car moves in a straight line. At time t (measured in seconds), its position (measured in meters) is

$$s(t) = t^2(3-t), 0 \leq t \leq 4.$$

a) [1.0 pt] Find its average velocity between $t = 0$ and $t = 4$.

b) [1.0 pt] Find its instantaneous velocity at any time t.

c) [1.0 pt] When the velocity is zero?

d) [1.0 pt] What is its maximum velocity?

e) [1.5 pt] Find the farthest distance from the car to the original position.

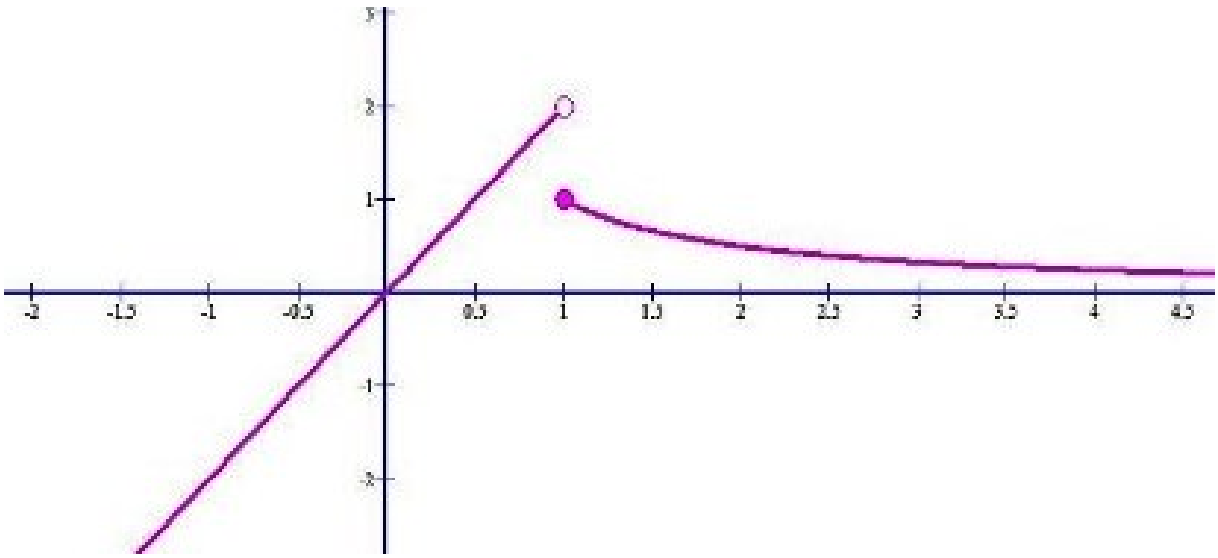
EDITED BY
Quang Sang PHAN

SOLUTION MIDTERM EXAM THE01001. CALCULUS 1
CODE 1: MID-TH1-1920, October 10,2019

Problem 1.

a) 1.0 pt When $c = 0$,

$$f(x) = \begin{cases} \frac{1}{x} & \text{for} \quad x \geq 1 \\ 2x & \text{if} \quad x < 1 \end{cases}$$



From the graph, we see that the function f is not continuous at $x = 1$.

b) 1.0 pt It is clear that the function is continuous for any $x \neq 1$.

At $x = 1$: $f(1) = 1$,

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{1}{x} = 1,$$
$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x + c) = 2 + c$$

The function is continuous at $x = 1$ iff

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = f(1) \Leftrightarrow 2 + c = 1 \Leftrightarrow c = -1$$

So that f is continuous for all $x \in (-\infty, +\infty)$ when $c = -1$.

Problem 2.

a) 1.0 pt The limiting population size is $\lim_{t \to +\infty} N(t) = \lim_{t \to +\infty} \frac{200t}{3+t} = \lim_{t \to +\infty} \frac{200}{\frac{3}{t}+1} = 200$.

b) 1.5 pt The growth rate is

$$\frac{dN}{dt} = \frac{600}{(3+t)^2}$$

The growth rate at time $t = 1$ is $\frac{600}{(3+1)^2} = 37.5$

$N(1) = 50$, and the per capital growth rate of the population at time $t = 1$ is

$$\frac{1}{N}\frac{dN}{dt} = \frac{37.5}{50} = 0.75$$

Problem 3. $s(t) = t^2(3-t), 0 \le t \le 4$.

a) [1.0 pt] The average velocity between $t = 0$ and $t = 4$ is

$$\frac{s(4) - s(0)}{4 - 0} = \frac{-16}{4} = -4$$

b) [1.0 pt] The instantaneous velocity at any time t is

$$v(t) = s'(t) = 6t - 3t^2, 0 \le t \le 4$$

c) [1.0 pt] The velocity is zero, i.e.

$$v(t) = s'(t) = 6t - 3t^2 = 0 \Leftrightarrow t = 0,\ t = 2$$

d) [1.0 pt] $v'(t) = 0 \Leftrightarrow 6 - 6t = 0 \Leftrightarrow t = 1$. We compare three values $v(0) = 0$, $v(1) = 3$, $v(4) = -24$. So the maximum velocity is 3 when $t = 1$.

e) [1.5 pt] The farthest distance from the car to the original position means $\mid s(t) - s(0) \mid = \mid s(t) \mid$ max.

$$s'(t) = 6t - 3t^2 = 0 \Leftrightarrow t = 0,\ t = 2$$

We compare three values $\mid s(0) \mid = 0$, $\mid s(2) \mid = 4$, and $\mid s(4) \mid = 16$. Hence the farthest distance is 16 when $t = 4$.

EDITED BY
Quang Sang PHAN