CODE 01: ME-MAT1092-01

**Advanced Mathematics - MIDTERM EXAM**

October 22, 2019

*Duration: 60 minutes*

**Problem 1.** [10 pts] The demand function of a good is

$$Q = 200 - 3P + 2Y + \frac{1}{3}A,$$

where $Q$, $P$, $Y$, $A$ denote quantity demanded, price, income and advertising expenditure respectively.

(a) [5 pts] Calculate the demand when $P = 4$, $Y = 30$, and $A = 6$.

(b) [5 pts] Is this good inferior or superior? Give a reason for your answer.

**Problem 2.** [15 pts] For a closed economy with no government intervention the consumption function is $C = 0.6Y + 250$ and planned investment is $I = 70$. Calculus the equilibrium level of national income, consumption, and savings.

**Problem 3.** [25 pts] The demand and supply functions of a good are given by

$$P = -2Q_D + 50, P = \frac{1}{2}Q_S + 25$$

where $P$, $Q_D$ and $Q_S$ denote the price, quantity demanded and quantity supplied respectively. The government decides to impose a tax $t$ per unit.

(a) [10 pts] Calculate the equilibrium quantity in term of $t$.

(b) [15 pts] Assume that equilibrium conditions prevail in the market. Give an expression for the government's total tax revenue in term of $t$, then find the value of $t$ which maximises it.

**Problem 4.** [25 pts] Assume that for a good, fixed costs are 10, variable costs are $Q + 3$ per unit and the demand function is

$$P = 60 - 0.5Q.$$

(a) [10 pts] Give expressions for the total cost and the marginal cost.

(b) [15 pts] Find the price elasticity of demand when the price is 30. Is the demand inelastic, unit elastic or elastic at this price?

**Problem 5.** [10 pts] A bank offers a return of 8% interest compounded quarterly. Find the future value of a principal of \$5000 after 4 years.

**Problem 6.** [15 pts] Determine the monthly repayments needed to repay a \$20000 loan that is paid back over 10 years when the interest rate is 8% compounded annually

Edited by Quang Sang Phan

CODE 02: ME-MAT1092-01

**Advanced Mathematics - MIDTERM EXAM**

October 22, 2019

*Duration: 60 minutes*

**Problem 1.** [10 pts] The demand function of a good is

$$Q = 250 - 3P - Y + \frac{1}{4}A,$$

where $Q$, $P$, $Y$, $A$ denote quantity demanded, price, income and advertising expenditure respectively.

(a) [5 pts] Calculate the demand when $P = 10$, $Y = 20$, and $A = 8$.

(b) [5 pts] Is this good inferior or superior? Give a reason for your answer.

**Problem 2.** [15 pts] For a closed economy with no government intervention the consumption function is $C = 0.7Y + 200$ and planned investment is $I = 46$. Calculus the equilibrium level of national income, consumption, and savings.

**Problem 3.** [25 pts] The demand and supply functions of a good are given by

$$P = -3Q_D + 50, P = \frac{1}{2}Q_S + 15$$

where $P$, $Q_D$ and $Q_S$ denote the price, quantity demanded and quantity supplied respectively. The government decides to impose a tax $t$ per unit.

(a) [10 pts] Calculate the equilibrium quantity in term of $t$.

(b) [15 pts] Assume that equilibrium conditions prevail in the market. Give an expression for the government's total tax revenue in term of $t$, then find the value of $t$ which maximises it.

**Problem 4.** [25 pts] Assume that for a good, fixed costs are 20, variable costs are $Q + 2$ per unit and the demand function is

$$P = 50 - 0.4Q.$$

(a) [10 pts] Give expressions for the total revenue and the marginal revenue.

(b) [15 pts] Find the price elasticity of demand when the price is 20. Is the demand inelastic, unit elastic or elastic at this price?

**Problem 5.** [10 pts] A bank offers a return of 10% interest compounded semi-annually. Find the future value of a principal of \$4000 after 5 years.

**Problem 6.** [15 pts] Determine the monthly repayments needed to repay a \$25000 loan that is paid back over 12 years when the interest rate is 8% compounded annually

Edited by Quang Sang Phan

CODE 01: SOL-ME-MAT1092-01

**Advanced Mathematics -SOLUTION MIDTERM EXAM**

October 22, 2019

*Duration: 60 minutes*

**Problem 1.** 10 pts

(a) 5 pts $Q = 200 - 3 \times 4 + 2 \times 30 + \frac{1}{3}6 = 250.$

(b) 5 pts The good is superior because if the income increases then the demand $Q$ rises (the coefficient in front of $Y$ is $2 > 0$).

**Problem 2.** 15 pts $C = 0.6Y + 250$, $I = 70$. In equilibrium, $Y = C + I$,

$$Y = 0.6Y + 250 + 70,$$

$$0.4Y = 320,$$

$$Y = 800.$$

Then $C = Y - I = 800 - 70 = 730$, $S = Y - C = 70$.

**Problem 3.** 25 pts

(a) 10 pts Because the tax is $t$ per unit, so the new supply function is

$$P - t = \frac{1}{2}Q_S + 25 \Leftrightarrow P = \frac{1}{2}Q_S + 25 + t.$$

In equilibrium $Q_D = Q_S = Q$, then we have

$$P = -2Q + 50 = \frac{1}{2}Q + 25 + t \Rightarrow Q = 10 - 0.4t.$$

(b) 15 pts In equilibrium the government's total tax revenue is

$$T = Q \times t = (10 - 0.4t)t = -0.4t^2 + 10t$$

$T$ is maximized when $t = \frac{-10}{2\times(-0.4)} = 12.5$ and the maximum value is $T = 62.5$.

**Problem 4.** 25 pts

(a) 10 pts In this problem $FC = 10$, $VC = Q + 3$. The total cost is

$$TC = FC + (VC)Q = 10 + (Q + 3)Q = Q^2 + 3Q + 10.$$

The marginal cost is

$$MC = \frac{d(TC)}{dQ} = 2Q + 3.$$

(b) 15 pts Since $P = 60 - 0.5Q \Rightarrow Q = 120 - 2P$.

When $P = 30$ then $Q = 60$, $\frac{dQ}{dP} = -2$,

$$E = -\frac{P}{Q} \times \frac{dQ}{dP} = -\frac{30}{60} \times (-2) = 1.$$

The demand is unit elastic at this price.

**Problem 5.** [10 pts] We have $P = \$5000$. The interest rate is 8% compounded quarterly so the interest rate in each quarter is $r\% = 2\%$, and the number of periods is $n = 4 \times 4 = 16$.

Let $q = 1 + r\% = 1 + 2\% = 1.02$. The future value is

$$S = Pq^n = 5000 \times 1.02^{16} = \$6863.93.$$

**Problem 6.** [15 pts] In this problem $L = \$20000$, $r\% = 8\%$, then $q := 1 + r\% = 1.08$, and $n = 10$. Let $a$ be the annually repayments. The loan is paid back over 10 years if

$$a = Lq^n \div \frac{q^n - 1}{q - 1} = 20000 \times 1.08^{10} \div \frac{1.08^{10} - 1}{1.08 - 1} = \$2980.59.$$

The monthly repayments are

$$\frac{a}{12} = \frac{2980.59}{12} = \$248.38.$$

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CODE 02: SOL-ME-MAT1092-01

**Advanced Mathematics - SOLUTION MIDTERM EXAM**

October 22, 2019

*Duration: 60 minutes*

**Problem 1.** 10 pts

(a) 5 pts $Q = 250 - 3 \times 10 - 20 + \frac{1}{4}8 = 202.$

(b) 5 pts The good is inferior because if the income $Y$ increases then the demand $Q$ decreases (the coefficient in front of $Y$ is $-1 < 0$).

**Problem 2.** 15 pts $C = 0.7Y + 200$, $I = 46$. In equilibrium: $Y = C + I$,

$$Y = 0.7Y + 200 + 46,$$
$$0.3Y = 246,$$
$$Y = 820.$$

Then $C = Y - I = 820 - 46 = 774$, $S = Y - C = 46$.

**Problem 3.** 25 pts

(a) 10 pts Because the a tax is $t$ per unit, so the new supply function is

$$P - t = \frac{1}{2}Q_S + 15 \Leftrightarrow P = \frac{1}{2}Q_S + 15 + t.$$

In equilibrium $Q_D = Q_S = Q$, then we have

$$P = -3Q + 50 = \frac{1}{2}Q + 15 + t \Rightarrow Q = 10 - \frac{2}{7}t.$$

(b) 15 pts In equilibrium the government's total tax revenue is

$$T = Q \times t = (10 - \frac{2}{7}t)t = -\frac{2}{7}t^2 + 10t$$

$T$ is maximized when $t = \frac{-10}{2\times(-\frac{2}{7})} = 17.5$ and the maximum value is $T = 87.5$.

**Problem 4.** 25 pts

(a) 10 pts The total revenue is

$$TR = PQ = (50 - 0.4Q)Q = -0.4Q^2 + 50Q$$

The marginal revenue is

$$MR = \frac{d(TR)}{dQ} = -0.8Q + 50.$$

(b) 15 pts Since $P = 50 - 0.4Q \Rightarrow Q = 125 - 2.5P$.

When $P = 20$ then $Q = 75$, $\frac{dQ}{dP} = -2.5$,

$$E = -\frac{P}{Q} \times \frac{dQ}{dP} = -\frac{20}{75} \times (-2.5) = 0.67.$$

The demand is inelastic at this price since $E = 0.67 < 1$.

**Problem 5.** [10 pts] We have $P = \$4000$. The interest rate is 10% compounded semi-annually so the interest rate in each half a year is $r\% = 5\%$, and the number of periods is $n = 2 \times 5 = 10$. Let $q = 1 + r\% = 1 + 5\% = 1.05$. The future value is

$$S = Pq^n = 4000 \times 1.05^{10} = \$6515.58.$$

**Problem 6.** [15 pts] In this problem $L = \$25000$, $r\% = 8\%$, then $q := 1 + r\% = 1.08$, and $n = 12$.

Let $a$ be the annually repayments. The loan is paid back over 10 years if

$$a = Lq^n \div \frac{q^n - 1}{q - 1} = 25000 \times 1.08^{12} \div \frac{1.08^{12} - 1}{1.08 - 1} = \$3317.38.$$

The monthly repayments are

$$\frac{a}{12} = \frac{3317.38}{12} = \$276.45.$$

Edited by Quang Sang Phan