CODE 01: ME-MAT1092-05

Student's full name:
Student's code:

Advanced Mathematics - MIDTERM EXAM

October 31, 2019

Duration: 60 minutes

Problem 1. $\boxed{\text{20 pts}}$ The demand and supply functions of a good are given by

$$P = -5Q_D + 70,\ P = 2Q_S + 8$$

where P, Q_D and Q_S denote the price, quantity demanded and quantity supplied respectively. If the government deducts, as tax, 20% of the market price of each good, determine the equilibrium price and quantity.

Problem 2. $\boxed{\text{20 pts}}$ Assume that for a good, fixed costs are 30, variable costs per unit are $Q + 3$, and the demand function is

$$P + 2Q = 50.$$

Give an expression for the profit in term of Q and then deduce the maximum profit.

Problem 3. $\boxed{\text{15 pts}}$ Table gives the annual rate of inflation during a 5-year period.

Year	2014	2015	2016	2017	2018
Inflation	3.3%	3%	3.5%	4%	4.2%

If a nominal house price at the end of 2014 was \$20.5 million, find the real house price adjusted to prices prevailing at the end of the year 2017. Round your answer to three significant figures.

Problem 4. $\boxed{\text{20 pts}}$ An individual saves \$3000 in a bank account at the beginning of each year for 8 years. Determine the total amount saved if the interest rate is 6% compounded

(a) annually;

(b) continuously.

Problem 5. $\boxed{\text{25 pts}}$ A project requires an initial outlay of \$10000 and produces returns of \$4000, \$5000 and \$6000 at the end of years 1, 2, and 3, respectively. If the market rate is 10%, find the present value of the cash flow and then the net present value of the project.

Edited by Quang Sang Phan

$\boxed{\text{CODE 02: ME-MAT1092-05}}$

Student's full name:
Student's code:

Advanced Mathematics - MIDTERM EXAM
October 31, 2019
Duration: 60 minutes

Problem 1. $\boxed{20 \text{ pts}}$ Assume that for a good, fixed costs are 7, variable costs per unit are $Q + 1$, and the demand function is

$$P + 0.5Q = 25.$$

Give an expression for the profit in term of Q and then deduce the maximum profit.

Problem 2. $\boxed{20 \text{ pts}}$ The demand and supply functions of a good are given by

$$P = -4Q_D + 80, \ P = 3Q_S + 6$$

where P, Q_D and Q_S denote the price, quantity demanded and quantity supplied respectively. If the government deducts, as tax, 25% of the market price of each good, determine the equilibrium price and quantity.

Problem 3. $\boxed{20 \text{ pts}}$ An individual saves \$5000 in a bank account at the beginning of each year for 6 years. Determine the total amount saved if the interest rate is 8% compounded

(a) annually;

(b) continuously.

Problem 4. $\boxed{15 \text{ pts}}$ Table gives the annual rate of inflation during a 5-year period.

Year	2014	2015	2016	2017	2018
Inflation	3%	3.5%	4%	3.8%	4.5%

If a nominal house price at the end of 2015 was \$18.5 million, find the real house price adjusted to prices prevailing at the end of the year 2018. Round your answer to three significant figures.

Problem 5. $\boxed{25 \text{ pts}}$ A project requires an initial outlay of \$15000 and produces returns of \$6000, \$7000 and \$8000 at the end of years 1, 2, and 3, respectively. If the market rate is 10%, find the present value of the cash flow and then the net present value of the project.

EDITED BY QUANG SANG PHAN

$\boxed{\text{CODE 03: ME-MAT1092-05}}$

Student's full name:
Student's code:

Advanced Mathematics - MIDTERM EXAM

October 31, 2019

Duration: 60 minutes

Problem 1. $\boxed{\text{20 pts}}$ Assume that for a good, fixed costs are 30, variable costs per unit are $Q+3$, and the demand function is

$$P + 2Q = 50.$$

Give an expression for the profit in term of Q and then deduce the maximum profit.

Problem 2. $\boxed{\text{20 pts}}$ The demand and supply functions of a good are given by

$$P = -5Q_D + 70, \;\; P = 2Q_S + 8$$

where P, Q_D and Q_S denote the price, quantity demanded and quantity supplied respectively. If the government deducts, as tax, 20% of the market price of each good, determine the equilibrium price and quantity.

Problem 3. $\boxed{\text{15 pts}}$ Table gives the annual rate of inflation during a 5-year period.

Year	2013	2014	2015	2016	2017
Inflation	3.5%	3.4%	3.8%	4%	4.5%

If a nominal house price at the end of 2013 was \$15.5 million, find the real house price adjusted to prices prevailing at the end of the year 2016. Round your answer to three significant figures.

Problem 4. $\boxed{\text{20 pts}}$ An individual saves \$6000 in a bank account at the beginning of each year for 5 years. Determine the total amount saved if the interest rate is 9% compounded

(a) annually;

(b) continuously.

Problem 5. $\boxed{\text{25 pts}}$ A project requires an initial outlay of \$20000 and produces returns of \$8000, \$9000 and \$10000 at the end of years 1, 2, and 3, respectively. If the market rate is 8%, find the present value of the cash flow and then the net present value of the project.

Edited by Quang Sang Phan

CODE 01: SOL-ME-MAT1092-05

Advanced Mathematics -SOLUTION MIDTERM EXAM

October 31, 2019

Duration: 60 minutes

Problem 1. 20 pts The new supply function is

$$P - 20\%P = 2Q_S + 8 \Leftrightarrow 0.8P = 2Q_S + 8 \Leftrightarrow P = 2.5Q_S + 10.$$

In equilibrium $Q_D = Q_S = Q$, then we have

$$P = -5Q + 70 = 2.5Q + 10 \Rightarrow Q = 8 \Rightarrow P = 30.$$

Problem 2. 20 pts In this problem $FC = 30$, $VC = Q + 3$.
Since

$$P + 2Q = 50 \Rightarrow P = 50 - 2Q.$$

The total revenue is

$$TR = PQ = (50 - 2Q)Q = -2Q^2 + 50Q.$$

The total cost is

$$TC = FC + (VC)Q = 30 + (Q + 3)Q = Q^2 + 3Q + 30.$$

The profit is

$$\pi = TR - TC = -2Q^2 + 50Q - (Q^2 + 3Q + 30) = -3Q^2 + 47Q - 30.$$

π is maximized when $Q = \frac{-b}{2a} = \frac{47}{6} = 7.83$. The corresponding maximum profit is $\pi_{max} = 154.08$.

Problem 3. 15 pts The overall scale is $q = 1.03 \times 1.035 \times 1.04$. The real house price is

$$P = 20.5 \times q = 20.5 \times 1.03 \times 1.035 \times 1.04 \simeq \$22.73.$$

Problem 4. 20 pts This is a sinking fund with $a = \$3000$. We use the formula for the total amount saved:

$$S = aq\frac{q^n - 1}{q - 1}$$

(a) annually: $r\% = 6\%$ then $q := 1 + r\% = 1.06$, and $n = 8$,

$$S = 3000 \times 1.06 \times \frac{1.06^8 - 1}{1.06 - 1} = \$31473.95.$$

(b) continuously: $q = e^{r\%} = e^{0.06}$, and $n = 8$

$$S = 3000 \times e^{0.06} \times \frac{e^{0.48} - 1}{e^{0.06} - 1} = \$31737.07.$$

Problem 5. $\boxed{\text{25 pts}}$ The initial outlay $P_0 = \$10000$; $r\% = 10\%$ then $q := 1 + r\% = 1.1$.

The present value of the project is

$$P = S_1 q^{-1} + S_2 q^{-2} + S_3 q^{-3} = 4000 \times 1.1^{-1} + 5000 \times 1.1^{-2} + 6000 \times 1.1^{-3} = 12276.484.$$

The net present value of the project is

$$NPV = P - P_0 = 12276.484 - 10000 = \$2276.484.$$

Edited by Quang Sang Phan

CODE 02: SOL-ME-MAT1092-05

Advanced Mathematics -SOLUTION MIDTERM EXAM

October 31, 2019

Duration: 60 minutes

Problem 1. $\boxed{\text{20 pts}}$ In this problem $FC = 7$, $VC = Q + 1$.

Since

$$P + 0.5Q = 25 \Rightarrow P = 25 - 0.5Q.$$

The total revenue is

$$TR = PQ = (25 - 0.5Q)Q = -0.5Q^2 + 25Q.$$

The total cost is

$$TC = FC + (VC)Q = 7 + (Q + 1)Q = Q^2 + Q + 7.$$

The profit is

$$\pi = TR - TC = -0.5Q^2 + 25Q - (Q^2 + Q + 7) = -1.5Q^2 + 24Q - 7.$$

π is maximized when $Q = \frac{-b}{2a} = 8$. The corresponding maximum profit is $\pi_{max} = 89$.

Problem 2. $\boxed{\text{20 pts}}$ The new supply function is

$$P - 25\%P = 3Q_S + 6 \Leftrightarrow 0.75P = 3Q_S + 6 \Leftrightarrow P = 4Q_S + 8.$$

In equilibrium $Q_D = Q_S = Q$, then we have

$$P = -4Q + 80 = 4Q + 8 \Rightarrow Q = 9 \Rightarrow P = 44.$$

Problem 3. $\boxed{\text{20 pts}}$ This is a sinking fund with $a = \$5000$. We use the formula for the total amount saved:

$$S = aq\frac{q^n - 1}{q - 1}$$

(a) annually: $r\% = 8\%$ then $q := 1 + r\% = 1.08$, and $n = 6$.

$$S = 5000 \times 1.08 \times \frac{1.08^6 - 1}{1.08 - 1} = \$39614.02.$$

(b) continuously: $q = e^{r\%} = e^{0.08}$, and $n = 6$,

$$S = 5000 \times e^{0.08} \times \frac{e^{0.48} - 1}{e^{0.08} - 1} = \$40065.37.$$

Problem 4. $\boxed{\text{15 pts}}$ The overall scale is $q = 1.04 \times 1.038 \times 1.045$. The real house price is

$$P = 18.5 \times q = 18.5 \times 1.04 \times 1.038 \times 1.045 \simeq \$20.87.$$

Problem 5. $\boxed{\text{25 pts}}$ The initial outlay $P_0 = \$15000$; $r\% = 10\%$ then $q := 1 + r\% = 1.1$.

The present value of the project is

$$P = S_1 q^{-1} + S_2 q^{-2} + S_3 q^{-3} = 6000 \times 1.1^{-1} + 7000 \times 1.1^{-2} + 8000 \times 1.1^{-3} = 17250.19.$$

The net present value of the project is

$$NPV = P - P_0 = 17250.19 - 15000 = \$2250.19.$$

Edited by Quang Sang Phan

$\boxed{\text{CODE 03: SOL-ME-MAT1092-05}}$

Advanced Mathematics -SOLUTION MIDTERM EXAM

October 31, 2019

Duration: 60 minutes

Problem 1. $\boxed{\text{20 pts}}$ In this problem $FC = 30$, $VC = Q + 3$.
Since

$$P + 2Q = 50 \Rightarrow P = 50 - 2Q.$$

The total revenue is

$$TR = PQ = (50 - 2Q)Q = -2Q^2 + 50Q.$$

The total cost is

$$TC = FC + (VC)Q = 30 + (Q + 3)Q = Q^2 + 3Q + 30.$$

The profit is

$$\pi = TR - TC = -2Q^2 + 50Q - (Q^2 + 3Q + 30) = -3Q^2 + 47Q - 30.$$

π is maximized when $Q = \frac{-b}{2a} = \frac{47}{6} = 7.83$. The corresponding maximum profit is $\pi_{max} = 154.08$.

Problem 2. $\boxed{\text{20 pts}}$ The new supply function is

$$P - 20\%P = 2Q_S + 8 \Leftrightarrow 0.8P = 2Q_S + 8 \Leftrightarrow P = 2.5Q_S + 10.$$

In equilibrium $Q_D = Q_S = Q$, then we have

$$P = -5Q + 70 = 2.5Q + 10 \Rightarrow Q = 8 \Rightarrow P = 30.$$

Problem 3. $\boxed{\text{15 pts}}$ The overall scale is $q = 1.034 \times 1.038 \times 1.04$. The real house price is

$$P = 15.5 \times q = 15.5 \times 1.034 \times 1.038 \times 1.04 \simeq \$17.30.$$

Problem 4. $\boxed{\text{20 pts}}$ This is a sinking fund with $a = \$6000$. We use the formula for the total amount saved:

$$S = aq\frac{q^n - 1}{q - 1}$$

(a) annually: $r\% = 9\%$ then $q := 1 + r\% = 1.09$, and $n = 5$.

$$S = 6000 \times 1.09 \times \frac{1.09^5 - 1}{1.09 - 1} = \$39140.01.$$

(b) continuously: $q = e^{r\%} = e^{0.09}$, and $n = 5$,

$$S = 6000 \times e^{0.09} \times \frac{e^{0.45} - 1}{e^{0.09} - 1} = \$39617.99.$$

Problem 5. $\boxed{\text{25 pts}}$ The initial outlay $P_0 = \$20000$; $r\% = 10\%$ then $q := 1 + r\% = 1.1$.

The present value of the project is

$$P = S_1 q^{-1} + S_2 q^{-2} + S_3 q^{-3} = 8000 \times 1.1^{-1} + 9000 \times 1.1^{-2} + 10000 \times 1.1^{-3} = 23061.779.$$

The net present value of the project is

$$NPV = P - P_0 = 23061.779 - 20000 = \$3061.779.$$

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