Chapter 6. Integration

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Content

Anti-derivative, or primitive $F'(x) = f(x)$

Example...

Indefinite integral

$$\int f(x)dx = F(x) + c, c \in \mathbb{R}$$

Some fundamental integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

$$\int e^{kx} dx = \frac{1}{k} e^{kx} + c,\ \ k \neq 0;\ \ \int \frac{1}{x} dx = \ln \mid x \mid + c$$

Examples and ppp 460-464

Some applications in economics

$$TR = \int MR \, dQ, \; TC = \int MC \, dQ,$$

$$C = \int MPC \, dY, \; C = \int MPS \, dY$$

Example: (page 464)

(a) A firm's marginal cost function is $MC = Q^2 + 2Q + 4$. Find the total cost function if the fixed costs are 100.

(b) The marginal revenue function of a monopolistic producer is $MR = 10 - 4Q$. Find the total revenue function and deduce the corresponding demand function.

Example: (page 464)

- Find an expression for the consumption function if the marginal propensity to consume is given by $MPC = 0.5 + \frac{0.1}{\sqrt{Y}}$ and consumption is 85 when income is 100.

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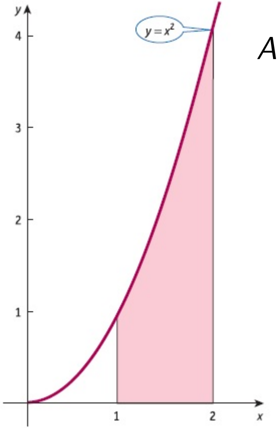
Key terms: page 466

*HW: Exercise 6.1, page 467
HW: Exercise 6.1, page 468, problems 2, 7, 12

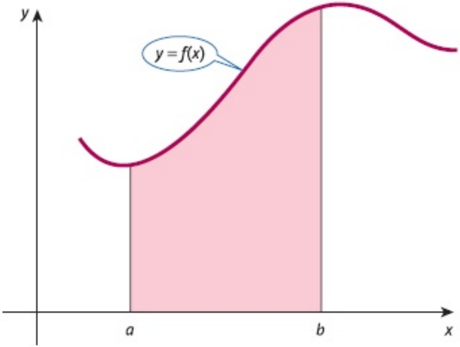
Area of a bounded region



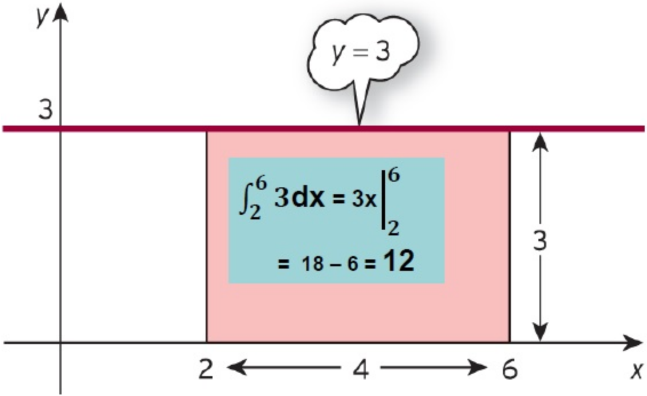
$$A = \int_1^2 x^2 dx = [\frac{x^3}{3}]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}.$$

Definite integral

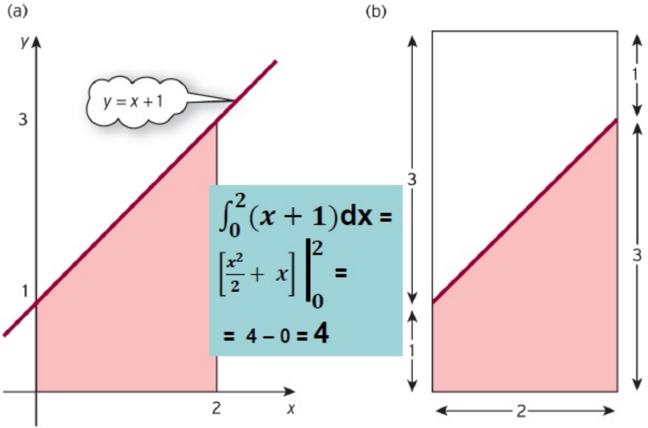
$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a) \tag{1}$$



Example



Example



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Example: evaluate the following integrals

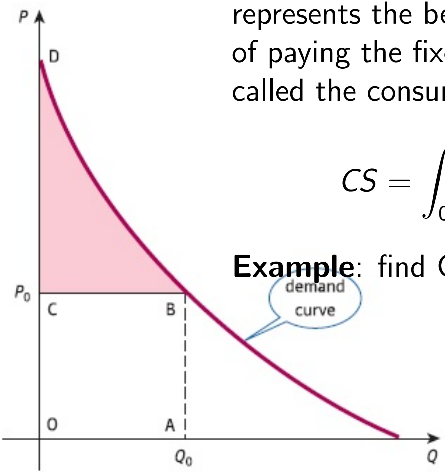
$$\int_0^1 (1 + 2x)dx, \quad \int_0^1 (x^2 + 2x)dx$$

Some applications of definite integration

- Consumer's surplus
- Producer's surplus
- Investment flow
- Discounting

Consumer's surplus: page 474

Let $P = f(Q)$ be the demand function. Assume that at $Q = Q_0$ the price $P = P_0$.



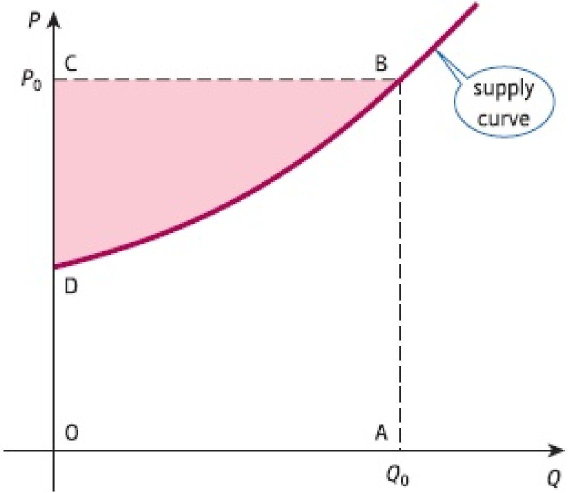
The shaded area BCD therefore represents the benefit to the consumer of paying the fixed price of P_0 and is called the consumer's surplus,denoted by CS.

$$CS = \int_0^{Q_0} f(Q)\, dQ - Q_0 P_0$$

Example: find CS at $Q = 5, P = 30 - 4Q$.

Producer's surplus: page 475

Let $P = g(Q)$ be the supply function. Assume that at $Q = Q_0$ the price $P = P_0$.



The shaded area BCD therefore represents the benefit to the producer of selling at the fixed price of P_0 and is called the producer's surplus, denoted by PS.

$$PS = Q_0 P_0 - \int_0^{Q_0} g(Q)\, dQ$$

Example: (page 476) given the demand function

$$P = 35 - Q_D^2,$$

and supply function

$$P = 3 + Q_S^2.$$

Find the producer's surplus assuming pure competition.

*ppp 477

Calculating the capital stock formation of Investment flow: page 477

Net investment flow $I(t)$ is the rate of change of capital stock K per time period (ex. measured in dollars per year),

$$I(t) = \frac{dK}{dt}$$

$K = K(t)$ is the amount of capital accumulated at time t as a result of this investment flow (measured in dollars),

$$K = \int I(t)dt.$$

In particular, the capital formation from t_1 to t_2 is

$$K(t_2) - K(t_1) = \int_{t_1}^{t_2} I(t)dt$$

Example (page 478): given $I(t) = 9000\sqrt{t}$

1. The capital formation between year 1 and year 4 is

$$\int_1^4 9000\sqrt{t}dt = [9000\frac{2}{3}t^{3/2}]_1^4 = \$42000$$

2. The number of years required before the capital stock first exceeds $ 100000:

$$\int_1^T 9000\sqrt{t}dt = 9000\frac{2}{3}T^{3/2} = 100000 \Rightarrow T = 6.5$$

The present value of a continuous revenue stream: page 479

If the fund is to provide a continuous revenue stream for n years at an annual rate of S dollars per year then the present value of the fund is

$$P = \int_0^n Se^{-r\%t}dt,$$

where $r\%$ is the nominal rate of interest.

Example (page 479): if the discount rate is 9% the present value of a continuous revenue stream of a $1000 a year for 5 years is

$$P = \int_0^5 Se^{-0.09t}dt = \$4026.35$$

* ppp 479

Key terms: page 480

HW: Exercise 6.2, page 480-481, problems 1-7;
Exercise 6.2*, page 481-482, problems 2-6, 8-9

THANK YOU VERY MUCH!