Chapter 8. Linear programming

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Content

Identify the region defined by a linear inequality

$$ax + by \;\Box\; c$$

Example: sketch the region defined by $2x + y < 4$

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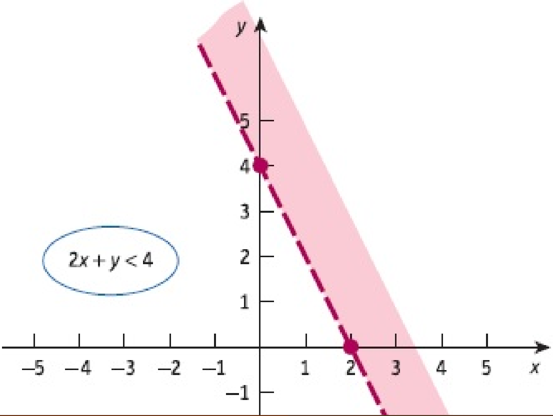
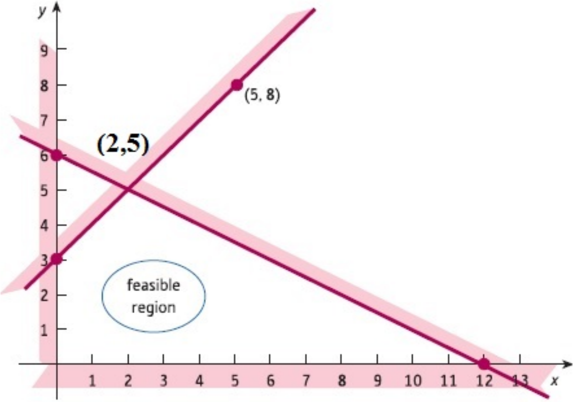


Sketch the feasible region defined by simultaneous linear inequalities

Example: (page 547) sketch the feasible region defined by $x + 2y \leq 12, \; -x + y \leq 3, \; x \geq 0, \; y \geq 0.$

Sketch the feasible region defined by simultaneous linear inequalities

Example: (page 547) sketch the feasible region defined by $x + 2y \leq 12,\ -x + y \leq 3,\ x \geq 0,\ y \geq 0.$



*ppp 549

Solve linear programming problems graphically

Example: (page 554) Minimize (or maximize) $-2x + y$ subject to the constraints

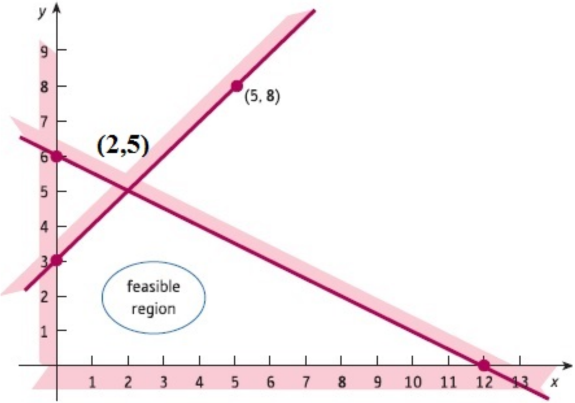
$$x + 2y \leq 12, \;\; -x + y \leq 3, \;\; x \geq 0, \;\; y \geq 0$$

Solve linear programming problems graphically

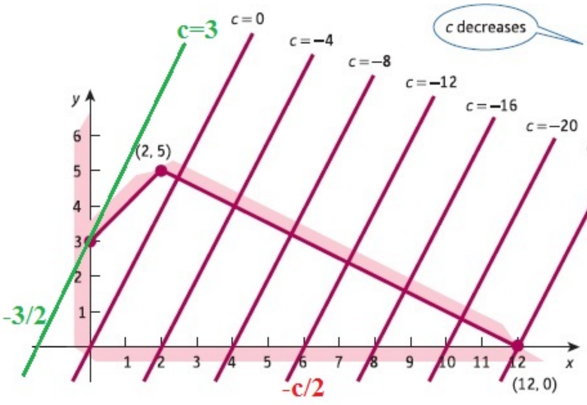
Example: (page 554) Minimize (or maximize) $-2x + y$ subject to the constraints

$$x + 2y \leq 12, \;\; -x + y \leq 3, \;\; x \geq 0, \;\; y \geq 0$$

Step1: sketch the feasible region defined by the constraints



Step 2: Let $c = -2x + y$ (d) and draw it



Note that the x-intercept of (d) is $-c/2$, and (d) passes through the feasible region iff

$$-3/2 \leq -c/2 \leq 12 \Leftrightarrow -24 \leq c \leq 3.$$

Minimum value of c is -24; Maximum value of c is 3

Remark: if the feasible region is bounded (a polygon), the optimal value of objective function is attained at one of the corners of the feasible region.

*We just compare the values of the objective function at the corners of the feasible region

Corner	Objective function
$(0, 0)$	$-2(0) + 0 = 0$
$(0, 3)$	$-2(0) + 3 = 3$
$(2, 5)$	$-2(2) + 5 = 1$
$(12, 0)$	$-2(12) + 0 = -24$

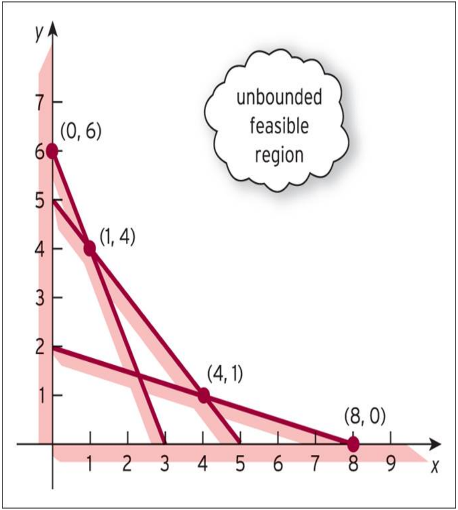
*Example page 552 and ppp 553

Optimisation with unbounded feasible region

Example: (page 555) Maximise $3x + 2y$ subject to the constraints

$$x + 4y \geq 8,\ \ x + y \geq 5,\ \ 2x + y \geq 6, x \geq 0,\ \ y \geq 0$$

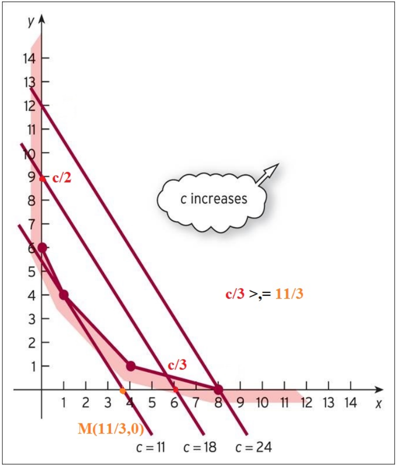
What can you say about the solution if this problem is one of minimisation rather than maximisation?



Corner	Objective function
(0, 6)	$3(0) + 2(6) = 12$
(1, 4)	$3(1) + 2(4) = 11$
(4, 1)	$3(4) + 2(1) = 14$
(8, 0)	$3(8) + 2(0) = 24$

* However the function $c = 3x + 2y$ does not reach the maximum value at the corner $(8, 0)$.

Sketch $c = 3x + 2y$ (d)



Note that the x-intercept of (d) is $c/3$, and (d) has common point with the feasible region iff

$$c/3 \geq 11/3 \Leftrightarrow c \geq 11$$

The objective function has not a maximum value. ($c_{min} = 11$)

Key terms: page 557

*HW: Exercise 8.1, page 558-559
HW: Exercise 8.1, page 559, problems 1-5

Example: A manufacture produces two kinds of good: A and B, for which demand exceeds capacity. The production costs for A and B are \$6 and \$3 correspondingly; and the corresponding selling prices are \$7 and \$4. The transport costs for each good of type A and B are 20 cents and 30 cents, respectively. Due to a bank loan limit, weekly production costs and weekly transport costs can not exceed \$2700 and \$120, respectively. How should the manufacture arrange production to maximize profit?

Steps:

- Identify the unknowns, called the decision variables (normally 2 variables, labeled x and y)
- Find an expression for the objective function in term of x and y.
- Write down all constraints on x and y
- Solve linear programming problems expressed in words, remembering to check that the answer makes sense.

We let

$$x = \text{ the number of good of type A to be made each weeek}$$

$$y = \text{ the number of good of type B to be made each weeek}$$

Maximise $x + y$ (profit) subject to the constraints

$$\begin{array}{rcll} 6x + 3y & \leq & 2700 & \text{(weekly production costs)} \\ 0.2x + 0.3y & \leq & 120 & \text{(weekly transport costs)} \\ x & \geq & 0 & \\ y & \geq & 0 & \end{array}$$

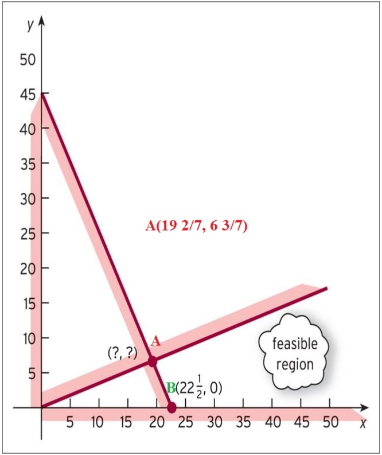
We find that the maximum weekly profit is \$405, which occurs when 375 goods of type A and 150 goods of type B are manufactured.

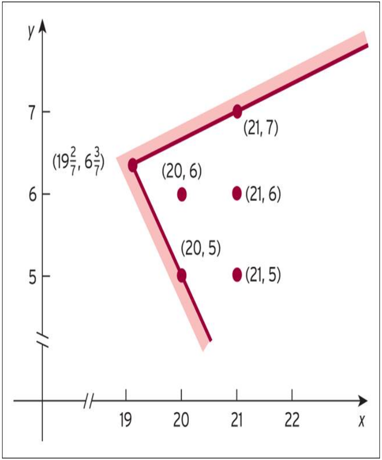
*ppp 564

Example: page 565 ; ppp 569

Minimise $800x + 320y$ subject to the constraints

$$\begin{array}{rcll} 40x + 20y & \geq & 900 & \text{(weekly worker-hours)} \\ y & \leq & x/3 & \text{(..1/3...)} \\ x & \geq & 0 & \\ y & \geq & 0 & \end{array}$$





Evaluate the objective function values at the above points
$\Rightarrow$ the minimum is \$17600 when $x = 20$ and $y = 5$.

Key terms: page 569

*HW: Exercise 8.2, page 569-570, problems 1-7
HW: Exercise 8.2, page 571-572, problems 2, 3, 5

Formal mathematics: page 574

THANK YOU VERY MUCH!